

# Universal Magic and Bimagic Squares of Orders 17 to 32 With Digits 1 and 8

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*The whole work is without use of any programming language.*

## Abstract

*This work brings **universal magic squares** of order 17 to 32. By **universal** we understand that the magic squares are **upside-down** and **mirror looking** independent of **magic square sums**. The work is for two digits as  $\{1,8\}$ . For the digits  $\{2,5\}$  and  $\{6,9\}$ , refer to Taneja [39, 40]. The same work can be extended for the digits  $\{0,1\}$  and  $\{0,8\}$ . Similar kind of work for orders 3-16 can be seen in Taneja [38]. The **block-wise** constructions for the magic squares of orders 18, 20, 21, 24, 25, 27, 28, 30 and 32 are also included. Moreover, in case of orders 25 and 32, we have **bimagic squares**, while for the case of order 24, it is **semi-bimagic square**. The whole work is without use of any **programming language**. It is just based on the number's combinations.*

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## 1 Introduction

In the previous paper [38], the author worked with three groups of numbers together for the magic squares of orders 3 to 16. In each group there are only two digits, i.e.,  $\{1, 8\}$ ,  $\{2, 5\}$  and  $\{6, 9\}$ . In the first and second cases, we have results as **upside-down** and **mirror looking**, while in the third case for the digits  $\{6, 9\}$ , the results are only **upside-down**. In the second group with the digits  $\{2, 5\}$ , the numbers 2 and 5 are written in digital form to have the conditions for **upside-down** and **mirror looking**. In the third case, 6 becomes 9 and 9 as 6 when we make  $180^\circ$  rotation. In some cases **palindromic-type magic squares** are also given. For more details refer author's work [38].

The whole work is based on combination of two digits. For example, 2-digits combinations of two letters  $a$  and  $b$  gives us 4 possibilities, i.e.,  $aa, ab, ba, bb$ . In the similar way considering two letters 3 by 3 combinations, there are 8 possibilities, i.e.,  $aaa, aab, aac, abb, acc, abc, bbb, ccc$ . Complete details of these combinations are given in Appendix 4.

This work is for the magic squares of orders 17 to 32 using the  $\{1, 8\}$ . These two numbers are always **upside-down** and **mirror looking**. The work for the digits  $\{2, 5\}$  and  $\{6, 9\}$  is given in Taneja [39, 40]. In some cases **block-wise** constructions are also given. These are for the magic squares of orders 18, 20, 21, 24, 25, 27, 28, 30 and 32. The magic squares are constructed in this work are **universal magic squares** i.e., **upside-down** and **mirror looking** independent of magic sums. The whole work is without use of any **programming language**. It is just with number's combinations. The study of magic squares of orders 33 to 64, and then from 65 to 128 shall be given in another works.

## 2 Universal Magic Squares With Digits 1 and 8

In the previous paper, we worked 2 by 2 combination of numbers in different ways. For example, in case of orders 3 and 4 we worked with 2-digits combinations. In case of orders 5 to 8 we worked with 3-digits combinations. In case of orders 9 to 16 we worked with 4-digits combinations. In this work we shall work with 5-digits combinations resulting in magic squares of order 17 to 32.



2.1 Magic Square of Order 17

**Example 2.1.** *The pandiagonal magic square of order 17 for the digits 1 and 8 is given below in two parts:*

17x17	pan	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173
	1118111181	1118888888	1181188811	1188888111	1811181118	1818118888	1888118181	1888811888	811111188
81818118173	8888811188	1111811118	1118188881	1118888188	1181181818	1188881111	1811118881	1818118111	1888111811
81818118173	8881111811	8888111181	8888888888	1111888811	1118188111	1118881118	1181118888	1188818181	1811111888
81818118173	8811111888	8818811188	8881111118	8888188881	8888888188	1111881818	1118181111	1118818881	1181118111
81818118173	8111818111	8181811811	8811111181	8818888888	8881188811	8888188111	8888881118	1111818888	1118118181
81818118173	1888818181	8111111888	8111811188	8181811118	8811188881	8818888188	8881181818	8888181111	8888818881
81818118173	1818118881	1888118111	1888811811	8111111181	8111888888	8181888811	8811188111	8818881118	8881118888
81818118173	1188818888	1811118181	1818111888	1888111188	1888811118	8111188881	8111888188	8181881818	8811181111
81818118173	1118881111	1181118881	1188818111	1811111811	1818111181	1888188888	1888888811	8111188111	8111881118
81818118173	1111881118	1118118888	1118818181	1181111888	1188811188	1811111118	1818188881	1888188188	1888881818
81818118173	8888181818	8888881111	1111818881	1118118111	1118811811	1181111181	1188888888	1811188811	1818188111
81818118173	8818888111	8881181118	8888118888	8888818181	1111811888	1118111188	1118811118	1181188881	1188888188
81818118173	8181888188	8811181818	8818881111	8881118881	8888118111	8888811811	1111811181	1118188888	1118888811
81818118173	8111188811	8111888111	8181881118	8811118888	8818818181	8881111888	8888111188	8888811118	1111888881
81818118173	1888188881	1888888188	8111181818	8111881111	8181818881	8811118111	8818811811	8881111181	8888188888
81818118173	1811188888	1818188811	1888188111	1888881118	8111118888	8111818181	8181811888	8811111188	8818811118
81818118173	1181111118	1188888881	1811188188	1818181818	1888181111	1888818881	8111118111	8111811811	8181811181
	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173
	1	2	3	4	5	6	7	8	9

81818118173	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173
8111811118	8181888881	8811188188	8818881818	8881181111	8888118881	8888818111	1111811811	81818118173
1888811181	8111188888	8111888811	8181888111	8811181118	8818818888	8881118181	8888111888	81818118173
1818111188	1888111118	1888888881	8111188188	8111881818	8181881111	8811118881	8818818111	81818118173
1188811811	1811111181	1818188888	1888188811	1888888111	8111181118	8111818888	8181818181	81818118173
1118811888	1181111188	1188811118	1811188881	1818188188	1888181818	1888881111	8111118881	81818118173
1111818111	1118111811	1118811181	1181188888	1188888811	1811188111	1818181118	1888118888	81818118173
8888118181	8888811888	1111811188	1118111118	1118888881	1181188188	1188881818	1811181111	81818118173
8818818881	8881118111	8888111811	8888811181	1111888888	1118188811	1118888111	1181181118	81818118173
8181818888	8811118181	8818811888	8881111188	8888111118	8888888881	1111888188	1118181818	81818118173
8111181111	8111818881	8181818111	8811111811	8818811181	8881188888	8888188811	8888888111	81818118173
1888181118	1888818888	8111118181	8111811888	8181811188	8811111118	8818888881	8881188188	81818118173
1811181818	1818181111	1888118881	1888818111	8111111811	8111811181	8181888888	8811188811	81818118173
1181188111	1188881118	1811118888	1818118181	1888111888	1888811188	8111111118	8111888881	81818118173
1118188188	1118881818	1181181111	1188818881	1811118111	1818111811	1888111181	1888888888	81818118173
8888888811	1111888111	1118181118	1118818888	1181118181	1188811888	1811111188	1818111118	81818118173
8881188881	8888188188	8888881818	1111881111	1118118881	1118818111	1181111811	1188811181	81818118173
8811188888	8818888811	8881188111	8888181118	8888818888	1111818181	1118111888	1118811188	81818118173
81818118173	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173	81818118173
10	11	12	13	14	15	16	17	

It is based on 2 by 2 combinations of 17 numbers

$$\{11118, 11181, 11188, 11811, 11888, 18111, 18181, 18881, 18888, \\ 81111, 81118, 81818, 88111, 88188, 88811, 88881, 88888\}$$

These 17 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{11181, 18111\}$ ,  $\{11188, 88111\}$ ,  $\{11888, 88811\}$ ,  $\{18111, 11181\}$  and  $\{18888, 88881\}$ . The numbers  $\{11811, 18181, 18881, 81118, 81818, 88188, 88888\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.1 is **universal pandiagonal magic square**. Its **magic sum** is

$$S_{17 \times 17} := 81818118173 = 100000 \times 818173 + 818173,$$

where

$$818173 := 11118 + 11181 + 11188 + 11811 + 11888 + 18111 + 18181 + 18881 + 18888 + \\ + 81111 + 81118 + 81818 + 88111 + 88188 + 88811 + 88881 + 8888.$$



2.2 Magic Square of Order 18

2.2.1 Blocks of Order 6

**Example 2.2.** *The magic square of order 18 for the digits 1 and 8 is given below in two parts:*

18x18	<u>mgc</u>							
1811111181	8188888811	8188888111	8188811888	1811111188	1811188818	1111111111	8888888118	8888881188
8188188818	1811811188	8188188111	1811811888	1811888811	8188111181	8811888888	1188111881	8811881188
8118188818	8118188811	1881811888	1881888111	8118111188	1881811181	8118888888	8118888118	1881118811
1881888818	1881811188	8118111888	8118188111	1881888811	8118111181	1881188888	1881111881	8118818811
1811811181	8188111188	1811888111	8188111888	8188188811	1811888818	1188111111	8811811881	1188181188
8188811181	1811188811	1811111888	1811188111	8188811188	8188888818	8888811111	1111188118	1111188111
1118111111	8881888118	8881881188	8881818811	1118111881	1118188888	1811118111	8188881881	8188881181
8881188888	1118811881	8881181188	1118818811	1118888118	8881111111	8188181888	1811818118	8188181181
8811188888	8811188118	1188818811	1188881188	8811111881	1188811111	8118181888	8118181881	1881818818
1188888888	1188811881	8811118811	8811181188	1188888118	8811111111	1881881888	1881818118	8118118818
1118811111	8881111881	1118881188	8881118811	8881188118	1118888888	1811818111	8188118118	1811881181
8881811111	1118188118	1118118811	1118181188	8881811881	8881888888	8188818111	1811181881	1811188118
1111118111	8888881881	8888881181	8888818818	1111118118	1111181888	1118111181	8881888811	8881888111
8811881888	1188118118	8811881181	1188118818	1188181881	8811818111	8881188818	1118811188	8881188111
8118881888	8118881881	1881118818	1881181181	8118818118	1881118111	8811188818	8811188811	1188811888
1881181888	1881118118	8118818818	8118881181	1881181881	8118818111	1188888818	1188811188	8811111888
1188118111	8811818118	1188181181	8811818818	8811881881	1188181888	1118811181	8881111188	1118888111
8888818111	1111181881	1111188118	1111181181	8888818118	8888881888	8881811181	1118188811	1118111888
89999999991	89999999991	89999999991	89999999991	89999999991	89999999991	89999999991	89999999991	89999999991
1	2	3	4	5	6	7	8	9

									89999999991
8888818811	1111111881	1111188888	1118118111	8881881881	8881881181	8881818818	1118118118	1118181888	89999999991
1188118811	1188188118	8811811111	8881181888	1118818118	8881181181	1118818818	1118881881	8881118111	89999999991
1881181188	8118811881	1881111111	8811181888	8811181881	1188818818	1188881181	8811118118	1188818111	89999999991
8118881188	1881188118	8118811111	1188881888	1188818118	8811118818	8811181181	1188881881	8811118111	89999999991
8811818811	8811888118	1188188888	1118818111	8881118118	1118881181	8881118818	8881181881	1118881888	89999999991
1111181188	8888811881	8888888888	8881818111	1118181881	1118118818	1118181181	8881818118	8881881888	89999999991
8188818818	1811118118	1811181888	1111111181	8888888811	8888888111	8888811888	1111111188	1111188818	89999999991
1811818818	1811881881	8188118111	8811888818	1188111188	8811888111	1188111888	1188188811	8811811181	89999999991
1881881181	8118118118	1881818111	8118888818	8118888811	1881111888	1881188111	8118811188	1881111181	89999999991
8118181181	1881881881	8118118111	1881188818	1881111188	8118811888	8118888111	1881188811	8118811181	89999999991
8188118818	8188181881	1811881888	1188111181	8811811188	1188188111	8811811888	8811888811	1188188818	89999999991
1811181181	8188818118	8188881888	8888811181	1111188811	1111111888	1111188111	8888811188	8888888818	89999999991
8881811888	1118111188	1118188818	1811111111	8188888118	8188881188	8188818811	1811111881	1811188888	89999999991
1118811888	1118888811	8881111181	8188188888	1811811881	8188181188	1811818811	1811888118	8188111111	89999999991
1188888111	8811111188	1188811181	8118188888	8118188118	1881818811	1881881188	8118111881	1881811111	89999999991
8811188111	1188888811	8811111181	1881888888	1881811881	8118118811	8118181188	1881888118	8118111111	89999999991
8881111888	8881188811	1118888818	1811811111	8188111881	1811881188	8188118811	8188188118	1811888888	89999999991
1118188111	8881811188	8881888818	8188811111	1811188118	1811118811	1811181188	8188811881	8188888888	89999999991
89999999991	89999999991	89999999991	89999999991	89999999991	89999999991	89999999991	89999999991	89999999991	89999999991
10	11	12	13	14	15	16	17	18	

It is based on 2 by 2 combinations of 18 numbers divided in three groups of 6 each having the same sum:

$$A := 11111 + 88888 + 18181 + 81818 + 11888 + 88111 = 299997$$

$$B := 11811 + 88188 + 11118 + 88881 + 18888 + 81111 = 299997$$

$$C := 18881 + 81118 + 11188 + 88811 + 11181 + 88818 = 299997.$$

In this case the magic sum of **upside-down magic square** is:

$$S_{18 \times 18} := 89999999991 = 100000 \times 899991 + 899991,$$

where

$$899991 := 11111 + 11118 + 11181 + 11188 + 11811 + 11888 + 18181 + 18881 + 18888 + \\ + 81111 + 81118 + 81818 + 88111 + 88188 + 88811 + 88818 + 88881 + 88888$$

Based on these three groups  $A, B$  and  $C$  nine magic squares of order 6 are constructed with equal magic sums  $S_{6 \times 6} := 29999999997$ . These nine magic square results in a magic square of order 18 given in Example 2.2.

Here we observe that even though the magic square is **upside-down** and **mirror looking**, i.e., it is **universal magic square**, but the magic sums may be different. Below are both the numbers, one those used in the magic square, and another when we make 180 degrees rotation or mirror looking case:

**Numbers used:**  $A := 11111 + 88888 + 18181 + 81818 + 11888 + 88111 = 299997$

$$B := 11811 + 88188 + 11118 + 88881 + 18888 + 81111 = 299997$$

$$C := 18881 + 81118 + 11188 + 88811 + 11181 + 88818 = 299997$$

**Up-side down:**  $A := 11111 + 88888 + 18181 + 81818 + 88811 + 11188 = 299997$

$$B := 11811 + 88188 + 81111 + 18888 + 88881 + 11118 = 299997$$

$$C := 18881 + 81118 + 88111 + 11888 + 18111 + 81888 = 299997$$

In this case the both the sums are same.

### 2.3 Magic Square of Order 19

**Example 2.3.** *The pandiagonal magic square of order 19 for the digits 1 and 8 is given below in two parts:*



19x19	pan	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802
	1111811118	1118188881	1118888188	1181181818	1181881118	1188818888	1811118181	1818111888	1888111811	1888811181
91181111802	8888111181	8888888888	1111888811	1118188111	1118881811	1181181111	1181818881	1188818111	1811111818	1818111188
91181111802	8818811188	8881111118	8888188881	8888888188	1111881818	1118181118	1118818888	1181118181	1181811888	1188811811
91181111802	8181811811	8811111181	8818888888	8881188811	8888188111	8888881811	1111881111	1118118881	1118818111	1181111818
91181111802	8111811818	8181111188	8181811118	8811188881	8818888188	8881181818	8888181118	8888818888	1111818181	1118111888
91181111802	1888811888	8111111811	8111811181	8181188888	8181888811	8811188111	8818881811	8881181111	8888118881	8888818111
91181111802	1818118111	1888111818	1888811188	8111111118	8111888881	8181188188	8181881818	8811181118	8818818888	8881118181
91181111802	1188818181	1811111888	1818111811	1888111181	1888888888	8111188811	8111888111	8181181811	8181881111	8811118881
91181111802	1181118881	1181818111	1188811818	1811111188	1818111118	1888188881	1888888188	8111181818	8111881118	8181118888
91181111802	1118118888	1118818181	1181111888	1181811811	1188811181	1811188888	1818188811	1888188111	1888881811	8111181111
91181111802	8888881111	1111818881	1118118111	1118811818	1181111188	1181811118	1188888881	1811188188	1818181818	1888181118
91181111802	8881181118	8888118888	8888818181	1111811888	1118111811	1118811181	1181188888	1181888811	1188888111	1811181811
91181111802	8811181811	8818881111	8881118881	8888118111	8888811818	1111811188	1118111118	1118888881	1181188188	1181881818
91181111802	8181181818	8181881118	8811118888	8818818181	8881111888	8888111811	8888811181	1111888888	1118188811	1118888111
91181111802	8111188111	8111881811	8181181111	8181818881	8811118111	8818811818	8881111188	8888111118	8888888881	1111888188
91181111802	1888188188	1888881818	8111181118	8111818888	8181118181	8181811888	8811111811	8818811181	8881188888	8888188811
91181111802	1811188811	1818188111	1888181811	1888881111	8111118881	8111818111	8181111818	8181811188	8811111118	8818888881
91181111802	1181888881	1188888188	1811181818	1818181118	1888118888	1888818181	8111111888	8111811811	8181111181	8181888888
91181111802	1118888888	1181188811	1181888111	1188881811	1811181111	1818118881	1888118111	1888811818	8111111188	8111811118
	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802
	1	2	3	4	5	6	7	8	9	10

91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802
8111188888	8111888811	8181188111	8181881811	8811181111	8818818881	8881118111	8888111818	8888811188	91181111802
1888111118	1888888881	8111188188	8111881818	8181181118	8181818888	8811118181	8818811888	8881111811	91181111802
1811111181	1818188888	1888188811	1888888111	8111181811	8111881111	8181118881	8181818111	8811111818	91181111802
1181811188	1188811118	1811188881	1818188188	1888181818	1888881118	8111118888	8111818181	8181111888	91181111802
1118811811	1181111181	1181888888	1188888811	1811188111	1818181811	1888181111	1888818881	8111118111	91181111802
1111811818	1118111188	1118811118	1181188881	1181888188	1188881818	1811181118	1818118888	1888118181	91181111802
8888111888	8888811811	1111811181	1118188888	1118888811	1181188111	1181881811	1188881111	1811118881	91181111802
8818818111	8881111818	8888111188	8888811118	1111888881	1118188188	1118881818	1181181118	1181818888	91181111802
8181818181	8811111888	8818811811	8881111181	8888188888	8888888811	1111888111	1118181811	1118881111	91181111802
8111818881	8181118111	8181811818	8811111188	8818811118	8881188881	8888188188	8888881818	1111881118	91181111802
1888818888	8111118181	8111811888	8181111811	8181811181	8811188888	8818888811	8881188111	8888181811	91181111802
1818181111	1888118881	1888818111	8111111818	8111811188	8181111118	8181888881	8811188188	8818881818	91181111802
1188881118	1811118888	1818118181	1888111888	1888811811	8111111181	8111888888	8181188811	8181888111	91181111802
1181181811	1181881111	1188818881	1811118111	1818111818	1888111188	1888811118	8111188881	8111888188	91181111802
1118181818	1118881118	1181118888	1181818181	1188811888	1811111811	1818111181	1888188888	1888888811	91181111802
8888888111	1111881811	1118181111	1118818881	1181118111	1181811818	1188811188	1811111118	1818188881	91181111802
8881188188	8888181818	8888881118	1111818888	1118118181	1118811888	1181111811	1181811181	1188888888	91181111802
8811188811	8818888111	8881181811	8888181111	8888818881	1111818111	1118111818	1118811188	1181111118	91181111802
8181188881	8181888188	8811181818	8818881118	8881118888	8888118181	8888811888	1111811811	1118111181	91181111802
91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802	91181111802
11	12	13	14	15	16	17	18	19	

It is based on 2 by 2 combinations of 19 numbers:

$$\{11118, 11181, 11188, 11811, 11818, 11888, 18111, 18181, 18881, 18888, \\ 81111, 81118, 81811, 81818, 88111, 88188, 88811, 88881, 88888\}$$

These 19 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{11181, 18111\}$ ,  $\{11188, 88111\}$ ,  $\{11888, 88811\}$ ,  $\{11818, 81811\}$  and  $\{18888, 88881\}$ . The numbers  $\{11811, 18181, 18881, 81118, 81818, 88188, 88888\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.3 is **universal pandiagonal magic square**. Its **magic sum** is

$$S_{19 \times 19} := 91181111802 = 100000 \times 911802 + 911802,$$

where

$$911802 := 11118 + 11181 + 11188 + 11811 + 11818 + 11888 + 18111 + 18181 + 18881 + 18888 + \\ + 81111 + 81118 + 81811 + 81818 + 88111 + 88188 + 88811 + 88881 + 88888.$$



2.4 Magic Square of Order 20

2.4.1 Blocks of Order 5

**Example 2.4.** *The magic square of order 20 for the digits 1 and 8 is given below in two parts:*

20x20	mgc								
1111811118	1881181111	1888818811	8111188881	8888118888	1118111188	1811188111	8118811881	8188888811	8881811888
8111118811	8888188881	1111818888	1881111118	1888881111	8188811881	8881888811	1118111888	1811111188	8118888111
1881118888	1888811118	8111181111	8888118811	1111888881	1811111888	8118811188	8188888111	8881811881	1118188811
8888181111	1111818811	1881188881	1888818888	8111111118	8881888111	1118111881	1811188811	8118811888	8188811188
1888888881	8111118888	8888111118	1111881111	1881118811	8118888811	8188811888	8881811188	1118188111	1811111881
1118811181	1188181888	1188818111	8811188818	8881181188	1181811818	1818888118	8181118188	8811888181	8818181811
8811118111	8881188818	1118881188	1188111181	1188881888	8811818188	8818188181	1181881811	1818811818	8181188118
1188181188	1188811181	8811181888	8881118111	1118888818	1818881811	8181111818	8811888118	8818118188	1181888181
8881181888	1118818111	1188188818	1188881188	8811111181	8818188118	1181818188	1818888181	8181181811	8811811818
1188888818	8811181188	8881111181	1118881888	1188118111	8181188181	8811881811	8818111818	1181888118	1818818188
1181811188	1818888111	8181111881	8811888811	8818111888	1118811118	1188181111	1188818811	8811188881	8881118888
8811811881	8818188811	1181811888	1818811188	8181188111	8811118811	8881188881	1118818888	1188111118	1188881111
1818811888	8181111188	8811888111	8818111881	1181888811	1188118888	1188811118	8811181111	8881118811	1118888881
8818188111	1181811881	1818888811	8181111888	8811811188	8881181111	1118818811	1188188881	1188818888	8811111118
8181188811	8811811888	8818111188	1181888111	1818811881	1188888881	8811118888	8881111118	1118881111	1188118811
1118111818	1811188118	8118818188	8188888181	8881881811	1111811181	1881181888	1888818111	8111188818	8888181188
8188818188	8881888181	1118181811	1811111818	8118888118	8111118111	8888188818	1111881188	1881111181	1888881888
1811181811	8118811818	8188888118	8881818188	1118188181	1881181188	1888811181	8111181888	8888118111	1111888818
8881888118	1118118188	1811188181	8118881811	8188811818	8888181888	1111818111	1881188818	1888881188	8111111181
8118888181	8188881811	8881811818	1118188118	1811118188	1888888818	8111181188	8888111181	1111881888	1881118111
9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990
1	2	3	4	5	6	7	8	9	10



										9999999990
1118811818	1188188118	1188818188	8811188181	8881181811	1181811181	1818881888	8181118111	8811888818	8818181188	9999999990
8811118188	8881188181	1118881811	1188111818	1188881118	8811818111	8818188818	1181881188	1818811181	8181181888	9999999990
1188181811	1188811818	8811188118	8881118188	1118888181	1818881188	8181111181	8811881888	8818118111	1181888818	9999999990
8881188118	1118818188	1188188181	1188881811	8811111818	8818181888	1181818111	1818888818	8181181188	8811811181	9999999990
1188888181	8811181811	8881111818	1118888118	1188118188	8181188818	8811881188	8818111181	1181881888	1818818111	9999999990
1111811188	1881188111	1888811881	8111188811	8888111888	1118111118	1811181111	8118818811	8188888881	8881818888	9999999990
8111111881	8888188811	1111811888	1881111188	1888888111	8188818811	8881888881	1118118888	1811111118	8118881111	9999999990
1881111888	1888811188	8111188111	8888111881	1111888811	1811118888	8118811118	8188881111	8881818811	1118188881	9999999990
8888188111	1111811881	1881188811	1888811888	8111111188	8881881111	1118118811	1811188881	8118818888	8188811118	9999999990
1888888811	8111111888	8888111188	1111888111	1881111881	8118888881	8188818888	8881811118	1118181111	1811118811	9999999990
1118111181	1811181888	8118818111	8188888818	8881881188	1111811818	1881188118	1888818188	8111188181	8888181811	9999999990
8188818111	8881888818	1118181188	1811111181	8118881888	8111118188	8888188181	1111881811	1881111818	1888888118	9999999990
1811181188	8118811181	8188881888	8881818111	1118188818	1881181811	1888811818	8111188118	8888118188	1111888181	9999999990
8881881888	1118118111	1811188818	8118881188	8188811181	8888188118	1111818188	1881188181	1888881811	8111111818	9999999990
8118888818	8188881188	8881811181	1118181888	1811118111	1888888181	8111181811	8888111818	1111888118	1881118188	9999999990
1181811118	1818881111	8181118811	8811888881	8818118888	1118811188	1188188111	1188811881	8811188811	8881111888	9999999990
8811818811	8818188881	1181818888	1818811118	8181181111	8811111881	8881188811	1118811888	1188111188	1188888111	9999999990
1818818888	8181111118	8811881111	8818118811	1181888881	1188111888	1188811188	8811188111	8881111881	1118888811	9999999990
8818181111	1181818811	1818888881	8181118888	8811811118	8881188111	1118811881	1188188811	1188811888	8811111188	9999999990
8181188881	8811818888	8818111118	1181881111	1818818811	1188888811	8811111888	8881111188	1118888111	1188111881	9999999990
9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990
11	12	13	14	15	16	17	18	19	20	

It is based on 2 by 2 combinations of 20 numbers:

$$\{11118, 11181, 11188, 11818, 11881, 11888, 18111, 18188, 18811, 18888, 81111, 81188, 81811, 81888, 88111, 88118, 88181, 88811, 88818, 88881\}$$

These 20 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{11181, 18111\}$ ,  $\{11188, 88111\}$ ,  $\{11818, 81811\}$ ,  $\{11881, 18811\}$ ,  $\{11888, 88811\}$ ,  $\{18188, 88181\}$ ,  $\{18888, 88881\}$ ,  $\{81188, 88118\}$  and  $\{81888, 88818\}$ . Thus the magic square given in Example 2.4 is **universal magic square**. Its **magic sum** is

$$S_{20 \times 20} := 9999999990 = 100000 \times 999990 + 999990,$$

where

$$999990 := 11118 + 11181 + 11188 + 11818 + 11881 + 11888 + 18111 + 18188 + 18811 + 18888 + 81111 + 81188 + 81811 + 81888 + 88111 + 88118 + 88181 + 88811 + 88818 + 88881.$$

The above magic square given in Example 2.4 is constructed based on each block of order 5. These blocks are pandiagonal with different magic sums resulting in a magic square of order 20.

2.4.2 Blocks of Order 4

**Example 2.5.** *The pandiagonal magic square of order 20 for the digits 1 and 8 is given below in two parts:*

20x20	pan	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990
	1188888111	8811188811	1118811188	8881111888	1188881111	8811188881	1118811118	8881118888	1188881888	8811188818
9999999990	1118811888	8881111188	1188888811	8811188111	1118818888	8881111118	1188888881	8811181111	1118818111	8881111181
9999999990	8881188811	1118888111	8811111888	1188811188	8881188881	1118881111	8811118888	1188811118	8881188818	1118881888
9999999990	8811111188	1188811888	8881188111	1118888811	8811111118	1188818888	8881181111	1118888881	8811111181	1188818111
9999999990	1888888111	8111188811	1111811188	8888111888	1888881111	8111188881	1111811118	8888118888	1888881888	8111188818
9999999990	1111811888	8888111188	1888888811	8111188111	1111818888	8888111118	1888888881	8111181111	1111818111	8888111181
9999999990	8888188811	1111888111	8111111888	1888811188	8888188881	1111881111	8111118888	1888811118	8888188818	1111881888
9999999990	8111111188	1888811888	8888188111	1111888811	8111111118	1888818888	8888181111	1111888881	8111111181	1888818111
9999999990	1811188111	8188888811	1118111188	8881811888	1811181111	8188888881	1118111118	8881818888	1811181888	8188888818
9999999990	1118111888	8881811188	1811188811	8188888111	1118118888	8881811118	1811188881	8188881111	1118118111	8881811181
9999999990	8881888811	1118188111	8188811888	1811111188	8881888881	1118181111	8188818888	1811111118	8881888818	1118181888
9999999990	8188811188	1811111888	8881888111	1118188811	8188811118	1811118888	8881881111	1118188881	8188811181	1811118111
9999999990	1818888111	8181188811	1181811188	8818111888	1818881111	8181188881	1181811118	8818118888	1818881888	8181188818
9999999990	1181811888	8818111188	1818888811	8181188111	1181818888	8818111118	1818888881	8181181111	1181818111	8818111181
9999999990	8818188811	1181888111	8181111888	1818811188	8818188881	1181881111	8181118888	1818811118	8818188818	1181881888
9999999990	8181111188	1818811888	8818188111	1181888811	8181111118	1818818888	8818181111	1181888881	8181111181	1818818111
9999999990	1881188111	8118888811	1188111188	8811811888	1881181111	8118888881	1188111118	8811818888	1881181888	8118888818
9999999990	1188111888	8811811188	1881188811	8118888111	1188118888	8811811118	1881188881	8118881111	1188118111	8811811181
9999999990	8811888811	1188188111	8118811888	1881111188	8811888881	1188181111	8118818888	1881111118	8811888818	1188181888
9999999990	8118811188	1881111888	8811888111	1188188811	8118811118	1881118888	8811881111	1188188881	8118811181	1881118111
	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990
	1	2	3	4	5	6	7	8	9	10



9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990
1118811181	8881118111	1188881811	8811188181	1118811818	8881118188	1188881188	8811188118	1118811881	8881118811	9999999990
1188888818	8811181888	1118818188	8881111818	1188888181	8811181811	1118818811	8881111881	1188888118	8811181188	9999999990
8811118111	1188811181	8881188181	1118881811	8811118188	1188811818	8881188118	1118881188	8811118811	1188811881	9999999990
8881181888	1118888818	8811111818	1188818188	8881181811	1118888181	8811111881	1188818811	8881181188	1118888118	9999999990
1111811181	8888118111	1888881811	8111188181	1111811818	8888118188	1888881188	8111188118	1111811881	8888118811	9999999990
1888888818	8111181888	1111818188	8888111818	1888888181	8111181811	1111818811	8888111881	1888888118	8111181188	9999999990
8111118111	1888811181	8888188181	1111881811	8111118188	1888811818	8888188118	1111881188	8111118811	1888811881	9999999990
8888181888	1111888818	8111111818	1888818188	8888181811	1111888181	8111111881	1888818811	8888181188	1111888118	9999999990
1118111181	8881818111	1811181811	8188888181	1118111818	8881818188	1811181188	8188888118	1118111881	8881818811	9999999990
1811188818	8188881888	1118118188	8881811818	1811188181	8188881811	1118118811	8881811881	1811188118	8188881188	9999999990
8188818111	1811111181	8881888181	1118181811	8188818188	1811111818	8881888118	1118181188	8188818811	1811111881	9999999990
8881881888	1118188818	8188811818	1811118188	8881881811	1118188181	8188811881	1811118811	8881881188	1118188118	9999999990
1181811181	8818118111	1818881811	8181188181	1181811818	8818118188	1818881188	8181188118	1181811881	8818118811	9999999990
1818888818	8181181888	1181818188	8818111818	1818888181	8181181811	1181818811	8818111881	1818888118	8181181188	9999999990
8181118111	1818811181	8818188181	1181881811	8181118188	1818811818	8818188118	1181881188	8181118811	1818811881	9999999990
8818181888	1181888818	8181111818	1818818188	8818181811	1181888181	8181111881	1818818811	8818181188	1181888118	9999999990
1188111181	8811818111	1881181811	8118888181	1188111818	8811818188	1881181188	8118888118	1188111881	8811818811	9999999990
1881188818	8118881888	1188118188	8811811818	1881188181	8118881811	1188118811	8811811881	1881188118	8118881188	9999999990
8118818111	1881111181	8811888181	1188181811	8118818188	1881111818	8811888118	1188181188	8118818811	1881111881	9999999990
8811881888	1188188818	8118811818	1881118188	8811881811	1188188181	8118811881	1881118811	8811881188	1188188118	9999999990
9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990	9999999990
11	12	13	14	15	16	17	18	19	20	

This magic square is also constructed with the same entries as of Example 2.4. As explained above these numbers lead us to a **universal magic square** with same magic sum, i.e.,

$$S_{20 \times 20} := 9999999990 = 100000 \times 999990 + 999990,$$

The magic square given in Example 2.5 is constructed based on blocks of order 4. These blocks are **pandiagonal** with **equal magic sum magic sums** resulting in a **pandiagonal magic square** of order 20. The blocks are constructed dividing the 20 numbers given in Example 2.4 in five groups of equal sums:

$$\begin{aligned}
 A &:= 11188 + 88111 + 11888 + 88811 = 199998 \\
 B &:= 11118 + 81111 + 18888 + 88881 = 199998 \\
 C &:= 11181 + 18111 + 88818 + 81888 = 199998 \\
 D &:= 11818 + 81811 + 18188 + 88181 = 199998 \\
 E &:= 11881 + 18811 + 81188 + 88118 = 199998.
 \end{aligned}$$



Based on these five  $A, B, C, D$  and  $E$ , the 25 **pandiagonal magic squares** of order 4 are constructed with equal **magic sums**:  $S_{4 \times 4} := 19999999998$ . These 25 magic square lead us to a **pandiagonal magic square** of order 20 given in Example 2.5.

2.5 Magic Square of Order 21

2.5.1 Without Block

**Example 2.6.** *The magic square of order 21 for the digits 1 and 8 is given below in two parts:*

21x21	<u>mgc</u>									
1111811118	8181811188	1181181818	1118811181	8881118811	8888181188	8888818888	8881881111	8188888811	8811888888	8818888818
1118881818	1118111181	1111811811	8181811118	8888881111	8881818888	8881181188	8888118811	8818888881	8811188818	8188888888
8181811811	1181111118	1118811188	1118181818	8881881188	8888818811	8888181111	8881118888	8811188888	8818888811	8811888881
1118111188	1111881818	8181811181	1181111811	8888118888	8881181111	8881818811	8888881188	8811888818	8188888881	8811188811
1881188811	8111188888	8118888818	1888888881	1188111881	8181818111	1818181818	1811111888	1111881888	1118888188	1181188111
8118888881	1888888818	1881188888	8111188811	1811181818	1188811888	1188118181	8181811881	1181188118	1118188111	1111888188
1888888888	8118888811	8111188881	1881188818	8181818181	1818111881	1811118111	1188881818	1118188188	1181181888	1118888118
8111188818	1881188881	1888888811	8118888888	1188818111	1188181818	8181811888	1818118181	1118888111	1111888118	1118181888
8881181888	8888188188	8888888111	8881888118	8188811118	8811811811	8818811181	8811111188	1881118811	8181881111	8118881818
8888888118	8881888111	8881188188	8888181888	8818811188	8811111181	8188811811	8811811118	8111181818	1888818888	1881181188
8881888188	8888881888	8888188118	8881188111	8811111181	8818811118	8811811188	8188811181	8181881188	8118818811	8111181111
8888188111	8881188118	8881881888	8888888188	8811811181	8188811188	8811111118	8818811811	1888881111	1881181818	8181818888
1181111181	1118811811	1118111118	1111811188	1818111888	1811118181	1188811881	1188118111	8118818888	8111181188	1888818811
1188118811	1811181188	1818118888	1188881111	1111888811	1118888888	1181188818	1118188881	8881111881	8888118181	8888811888
1818181111	1188818888	1188181188	1811118811	1181188881	1118188818	1111888888	1118888811	8888818111	8881811888	8881118181
1188881188	1818118811	1811181111	1188118888	1118188888	1181188811	1118888881	1111888818	8881818181	8888811881	8888118111
1811118888	1188181111	1188818811	1818181188	1118888818	1111888881	1118188811	1181188888	8888111888	8881118111	8881811881
8188811881	8811818181	8818811888	8811118111	1881181888	8111188188	8118888111	1888888118	1188111118	1811111811	1818111181
8818818111	8811111888	8188818181	8811811881	8118888118	1888888111	1881188188	8111181888	1818111188	1188811181	1188111811
8811118181	8818811881	8811818111	8188811888	1888888188	8118881888	8111188118	1881188111	1188811811	1818111118	1811111188
8811811888	8188818111	8811111881	8818818181	8111188111	1881188118	1888881888	8118888188	1811111181	1188111188	1188811118
108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878
1	2	3	4	5	6	7	8	9	10	11

										10888888878
8811188881	1118111811	1881111881	8111118181	8118811888	1888818111	1188181888	1811188188	1818188111	1188888118	10888888878
8811888811	1181111188	8118818111	1888811888	1881118181	8111111881	1818188118	1188888111	1188188188	1811181888	10888888878
8188888818	1111811181	1888818181	8118811881	8111118111	1881111888	1188888188	1818181888	1811188118	1188188111	10888888878
8818888888	1118811118	8111111888	1881118111	1888811881	8118818181	1811188111	1188188118	1188881888	1818188188	10888888878
1118188118	1188818181	8881111118	8888111811	8888811181	8881811188	8188818811	8811881188	8818818888	8811181111	10888888878
1118881888	1818181111	8888811188	8881811181	8881111811	8888111118	8818881111	8811118888	8188881188	8811818811	10888888878
1111888111	1188111888	8881811811	8888811118	8888111188	8881111181	8811181188	8818818811	8811881111	8188818888	10888888878
1181188188	1811111881	8888111181	8881111188	8881811118	8888811811	8811818888	8188881111	8811118811	8818881188	10888888878
8111118888	1888881188	1188188811	1811188888	1818188818	1188888881	1111811881	1118818181	1181111888	1118118111	10888888878
8181818811	8118881111	1818188881	1188888818	1188188888	1811188811	1181118111	1118111888	1111818181	1118811881	10888888878
1888881818	1881118888	1188888888	1818188811	1811188881	1188188818	1118118181	1181111881	1118818111	1111811888	10888888878
8118881188	8111118811	1811188818	1188188881	1188888811	1818188888	1118811888	1111818111	1118111881	1181118181	10888888878
1881181111	8181881818	8818888111	8811888188	8811181888	8188888118	8888888818	8888188888	8881888811	8881188881	10888888878
8881818111	8811188188	8188881888	8181888118	8818881818	8811888111	1881111118	8111111811	8118811181	1888811188	10888888878
8888111881	8818888118	8811881818	8811188111	8188888188	8181881888	8118811188	1888811181	1881111811	8111111118	10888888878
8881111888	8188888111	8181888188	8818881888	8811888118	8811181818	1888811811	8118811118	8111111188	1881111181	10888888878
8888818181	8811881888	8811188118	8188881818	8181888111	8818888188	8111111181	1881111188	1888811118	8118811811	10888888878
1188811188	8881888888	1111818811	1118881188	1181118888	1118181111	8881188811	8181888881	8888881818	8888188818	10888888878
1811111118	8888888881	1181181111	1118118888	1111881188	1118818811	8888181818	8881888818	8881188888	8181888811	10888888878
1188111181	8881188818	1118181188	1181118811	1118881111	1111818888	8181888888	8888888811	8888188881	8881881818	10888888878
1818111811	8888188811	1118818888	1111881111	1118118811	1181181188	8881888881	8881181818	8181888818	8888888888	10888888878
10888888878	10888888878	10888888878	10888888878	10888888878	10888888878	10888888878	10888888878	10888888878	10888888878	10888888878
12	13	14	15	16	17	18	19	20	21	

It is based on 2 by 2 combinations of 21 numbers:

$$\{11118, 11181, 11188, 11811, 11881, 11888, 18111, 18181, 18811, 18888, 81111, 81188, 81818, 81888, 88111, 88118, 88188, 88811, 88818, 88881, 88888\}.$$

These 21 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{11188, 88111\}$ ,  $\{11888, 88811\}$ ,  $\{11181, 18111\}$ ,  $\{11881, 18811\}$ ,  $\{81188, 88118\}$ ,  $\{18888, 88881\}$  and  $\{81888, 88818\}$ . The numbers  $\{11811, 18181, 81818, 88188, 88888\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.6 is **universal magic square**. Its **magic sum** is

$$S_{21 \times 21} := 108888888878 = 100000 \times 1088878 + 1088878,$$

where



$$1088878 := 11118 + 11181 + 11188 + 11811 + 11881 + 11888 + 18111 + 18181 + 18811 + 18888 + 81111 +$$
$$+ 81188 + 81818 + 81888 + 88111 + 88118 + 88188 + 88811 + 88818 + 88881 + 88888.$$

2.5.2 Block of Order 7

**Example 2.7.** *The semi-magic square of order 21 for the digits 1 and 8 is given below in two parts:*

21x21	semi-mgc									
111811118	1188111888	1818118111	8111181188	8188881818	8818888811	8888188818	1118811188	1181111811	1881118811	1888818888
8818881818	8888188811	1118188818	1188111118	1818111888	8111118111	8188881188	8811888111	8888888118	1118888888	1181111188
8111111888	8188818111	8818881188	8888181818	1118188811	1188188818	1818111118	1888811811	8811118811	8811818888	8888888111
1188188811	1818188818	8111111118	8188811888	8818818111	8888181188	1118181818	1181188118	1881188888	1888811188	8811111811
8888118111	1118181188	1188181818	1818188811	8111188818	8188811118	8818811888	8888818811	1118818888	1181188111	1881188118
8188888818	8818811118	8888111888	1118118111	1188181188	1818181818	8111188811	8811188888	8811811188	8888811811	1118818811
1818181188	8111181818	8188888811	8818888818	8888111118	1118111888	1188118111	1881118888	1888888111	8811188118	8811888888
1111811188	1188811811	1811118811	8118818888	8181888111	8881188118	8881888888	1118111181	1188111881	1818118181	8111181111
8881188111	8881888118	1111888888	1188811188	1811111811	8118818811	8181818888	8818881888	8888188188	1118188881	1188111181
8118811811	8181818811	8881118888	8881888111	1111888118	1188888888	1811111188	8111111881	8188818181	8818881111	8888181888
1188888118	1811188888	8118811188	8181811811	8881118811	8881818888	1111888111	1188188188	1818188881	8111111181	8188811881
8881818811	1111818888	1188888111	1811188118	8118888888	8181811188	8881111811	8888118181	1118181111	1188181888	1818188188
8181888888	8881111188	8881811811	1111818811	1188818888	1811188111	8118888118	8188888881	8818811181	8888111881	1118118181
1811118888	8118888111	8181888118	8881188888	8881811188	1111811811	1188818811	1818181111	8111181888	8188888188	8818888881
1118811181	1181111881	1881118181	1888881111	8811181888	8811888188	8888888881	1111811118	1188811888	1811118111	8118881188
8811881888	8888888188	1118888881	1181111181	1881111881	1888818181	8811181111	8881181818	8881888811	1111888818	1188811118
1888811881	8811118181	8811881111	8888881888	1118888188	1181188881	1881111181	8118811888	8181818111	8881181188	8881881818
1181188188	1881188881	1888811181	8811111881	8811818181	8888881111	1118881888	1188888811	1811188818	8118811118	8181811888
8888818181	1118881111	1181181888	1881188188	1888888881	8811111181	8811811881	8881818111	1111881188	1188881818	1811188811
8811188881	8811811181	8888811881	1118818181	1181181111	1881181888	1888888188	8181888818	8881111118	8881811888	1111818111
1881181111	1888881888	8811188188	8811888881	8888811181	1118811881	1181118181	1811181188	8118881818	8181888811	8881188818
108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878
1	2	3	4	5	6	7	8	9	10	11



										108888943933
881188111	8811888118	8888888888	1111811181	1188811881	1811118181	8118881111	8181881888	8881188188	8881888881	108888888878
1881111811	1888818811	8811118888	8881181888	8881888188	1111888881	1188811181	1811111881	8118818181	8181881111	108888888878
1118888118	1181188888	1881111188	8118811881	8181818181	8881181111	8881881888	1111888188	1188888881	1811111181	108888888878
8811818811	8888818888	1118888111	1188888188	1811188881	8118811181	8181811881	8881118181	8881881111	1111881888	108888888878
1888888888	8811111188	8811811811	8881818181	1111881111	1188881888	1811188188	8118888881	8181811181	8881111881	108888888878
1181118888	1881188111	1888888118	8181888881	8881111181	8881811881	1111818181	1188881111	1811181888	8118888188	108888888878
8888811188	1118811811	1181118811	1811181111	8118881888	8181888188	8881188881	8881811181	1111811881	1188818181	108888888878
8188881888	8818888188	8888188881	1118811118	1181111888	1881118111	1888881188	8811181818	8811888811	8888888818	108888888878
1818111881	8111118181	8188881111	8811881818	8888888811	1118888818	1181111118	1881111888	1888818111	8811181188	108888888878
1118188188	1188188881	1818111181	1888811888	8811118111	8811881188	8888881818	1118888811	1181188818	1881111118	108888888878
8818818181	8888181111	1118181888	1181188811	1881188818	1888811118	8811111888	8811818111	8888881188	1118881818	108888888878
8111188881	8188811181	8818811881	8888818111	1118881188	1181181818	1881188811	1888888818	8811111118	8811811888	108888888878
1188181111	1818181888	8111188188	8811188818	8811811118	8888811888	1118818111	1181181188	1881181818	1888888811	108888888878
8888111181	1118111881	1188118181	1881181188	1888881818	8811188811	8811888818	8888811118	1118811888	1181118111	108888888878
8181881818	8881188811	8881888818	1118111188	1188111811	1818118811	8111118888	8188888111	8818888118	8888188888	108888888878
1811111888	8118818111	8181881188	8818888111	8888188118	1118188888	1188111188	1818111811	8111118811	8188818888	108888888878
1111888811	1188888818	1811111118	8111111811	8188818811	8818818888	8888188111	1118188118	1188188888	1818111188	108888888878
8881118111	8881881188	1111881818	1188188118	1818188888	8111111188	8188811811	8818818811	8888118888	1118188111	108888888878
8118888818	8181811118	8881111888	8888118811	1118118888	1188188111	1818188118	8111188888	8188811188	8818811811	108888888878
1188881188	1811181818	8118888811	8188888888	8818811188	8888111811	1118118811	1188118888	1818188111	8111188118	108888888878
8881811118	1111811888	1188818111	1818118888	8111188111	8188888118	8818888888	8888111188	1118111811	1188118811	108888888878
108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	114394388878
12	13	14	15	16	17	18	19	20	21	

This magic square is constructed with the same entries as of Example 2.7. As explained above these numbers lead us to a **universal semi-magic square** with **semi-magic sum** i.e.,

$$S_{21 \times 21} := 108888888878 = 100000 \times 1088878 + 1088878.$$

By the **semi-magic square**, it is understood that the sum of rows and columns is the same, while the principal diagonals sum is different. The magic square given in Example 2.7 is constructed based on blocks of order 7. These 9 blocks of order 7 are **pandiagonal magic squares** with different magic sums resulting in a **semi-magic square** of order 21.

### 2.5.3 Block of Order 3

**Example 2.8.** *The semi-magic square of order 21 for the digits 1 and 8 is given below in two parts:*

21x21	semi-mgc									
8118181888	1811118111	8188881181	8188111118	8881188811	1111881881	1881888818	1188811888	8881818818	8111111181	8888188881
8188818111	8118181181	1811181888	1111888811	8188181881	8881111118	8881811888	1881818818	1188888818	1118188881	8111181111
1811181181	8188881888	8118118111	8881181881	1111811118	8188188811	1188818818	8881888818	1881811888	8888181111	1118111181
8181888888	8818811811	1118818181	1888811188	1811888188	8811181818	8118188111	1811118118	8188818888	8188181888	8881118111
1118811811	8181818181	8818888888	8811188188	1888881818	1811811188	8188818118	8118118888	1811881111	1111818111	8188181181
8818818181	1118888888	8181811811	1811881818	8811111188	1888888188	1811118888	8188888111	8118118118	8881181181	1111881888
8111111118	8888188811	1118181881	1818188818	1181111888	8888818818	8181811181	8818888881	1118881111	1888888888	1811811811
1118188811	8111181881	8888111118	8888811888	1818118818	1181188818	1118888881	8181881111	8818811181	8811111811	1888818181
8888181881	1118111118	8111188811	1181118818	8888888818	1818111888	8818881111	1118811181	8181888881	1811818181	8811188888
8188111188	8881188188	1111881818	1881888111	1188818118	8881818888	8111181888	8888181111	1118181181	1818111118	1181188811
1111888188	8188181818	8881111188	8881818118	1881818888	1188888111	1118118111	8111181181	8888181888	8888888811	1818181881
8881181818	1111811188	8188188188	1188818888	8881888111	1881818118	8888181181	1118181888	8111181111	1181181881	8888811118
1888888818	1811811888	8811118818	8118111181	1811188881	8188881111	8188188888	8881111811	1111818181	1881811188	1188888188
8811111888	1888818818	1811888818	8188888881	8118181111	1811111181	1111811181	8188118181	8881188888	8881888188	1881881818
1811818818	8811188818	1888811888	1811181111	8188811181	8118188881	8881118181	1111888888	8188111811	1188881818	8881811188
1818188111	1181118118	8888818888	8181881888	8818818111	1118881181	1888811118	1811888811	8811181881	8118188818	1811111888
8888818118	1818118888	1181188111	1118818111	8181881181	8818881888	8811188811	1888881881	1811811118	8188811888	8118118818
1181118888	8888888111	1818118118	8818881181	1118881888	8181818111	1811881881	8811111118	1888888811	1811118818	8188888818
1881811181	1188888881	8881881111	8111188888	8888111811	1118118181	1818111188	1181188188	8888881818	8181888111	8818818118
8881888881	1881881111	1188811181	1118111811	8111118181	8888188888	8888888188	1818181818	1181111188	1118818118	8181818888
1188881111	8881811181	1881888881	8888118181	1118188888	8111111811	1181181818	8888811188	1818188188	8818818888	1118888111
108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878
1	2	3	4	5	6	7	8	9	10	11



										108888945634
111818111	181818888	118111181	888881818	818181188	8818888188	1118881818	188888811	181181818	8811118888	108888888878
888811181	888881181	181818181	1181188888	1118888188	8181881818	8818811188	8811118118	1888818888	181188811	108888888878
8111188881	1181118181	8888888888	1818111811	8818881818	1118811188	8181888188	1811818888	8811188111	1888818118	108888888878
1111881181	1881811118	1188888811	8881881881	8111188818	8888111888	1118118818	1818111181	1181188881	8888881111	108888888878
8881181888	8881888811	1881881881	1188811118	1118111888	8111118818	8888188818	8888888881	1818181111	1181111181	108888888878
8188118111	1188881881	8881811118	1881888811	8888118818	1118188818	8111111888	1181181111	8888811181	1818188881	108888888878
8811118181	8118111188	1811188188	8188881818	8188188111	8881118118	1111818888	1881881888	1188818111	8881881181	108888888878
1811888888	8188888188	8118181818	1811111188	1111818118	8188118888	8881188111	8881818111	1881881181	1188881888	108888888878
1888811811	1811181818	8188811188	8118188188	8881118888	1111888111	8188118118	1188881181	8881881888	1881818111	108888888878
8888881881	8181888818	8818811888	1118818818	1888811181	1811888881	8811181111	8118188888	1811111811	8188818181	108888888878
1181111118	1118811888	8181818818	8818888818	8811188881	1888881111	1811811181	8188811811	8118118181	1811188888	108888888878
1818188811	8818818818	1118888818	8181811888	1811881111	8811111181	1888888881	1811118181	8188888888	8118111811	108888888878
8881881818	8111188111	8888118118	1118118888	1818181888	1181118111	8888881181	8181811118	8818888811	1118881881	108888888878
1188811188	1118118118	8111118888	8888188111	8888818111	1818181181	1181181888	1118888811	8181881881	8818811118	108888888878
1881888188	8888118888	1118188111	8111118118	1181181181	8888881888	1818118111	8818881881	1118811118	8181888811	108888888878
8188818818	8188111181	8881188881	1111881111	1881888888	1188811811	8881818181	8111111188	8888188188	1118181818	108888888878
1811188818	1111888881	8188181111	8881111181	8881811811	1881818181	1188888888	1118188188	8111181818	8888111188	108888888878
8118111888	8881181111	1111811181	8188188881	1188818181	8881888888	1881811811	8888181818	1118111188	8111188188	108888888878
1118818888	1888881888	1811818111	8811181181	8118111118	1811188811	8188881881	8188188818	8881111888	1111818818	108888888878
8818888111	8811118111	1888881181	1811881888	8188888811	8118181881	1811111118	1111811888	8188118818	8881188818	108888888878
8181818118	1811881181	8811181888	1888818111	1811181881	8188811118	8118188811	8881118818	1111888818	8188111888	108888888878
108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	108888888878	114564488878
12	13	14	15	16	17	18	19	20	21	

This magic square is constructed with the same entries as of Example 2.6. In this case, these numbers lead us to a **universal semi-magic square** with **semi-magic sum** i.e.,

$$S_{21 \times 21} := 108888888878 = 100000 \times 1088878 + 1088878.$$

The magic square given in Example 2.8 is constructed based on blocks of order 3. These 21 blocks of order 3 are also **semi-magic squares** with different **semi-magic sums**. These 21 blocks of order 3 lead us to a **semi-magic square** of order 21 given in Example 2.8

**Remark 2.1.** *The aim of making magic square of order 21 given in Example 2.6 is due to the fact that the other two magic squares given in Examples 2.7 and 2.8 are **semi-magic squares**. It is constructed based on the **self orthogonal diagonalize Latin squares**. For examples of SODLS refer to H. White [3, 4].*



2.6 Magic Square of Order 22

**Example 2.9.** *The magic square of order 22 for the digits 1 and 8 is given below in two parts:*

22x22	<a href="#">mgc</a>									
1111111111	8811118888	8181881111	1888181818	8188111118	1818188811	8811818118	8111188111	1888888118	1111811811	8888888881
1888888111	1111811118	8188118888	8118881111	1111181188	8181811188	1881888188	8811118181	8111181881	1118811888	8888188811
8111181818	1888881881	1118811188	8181818888	8888881111	1111888888	8118811811	1888188118	8188118818	1181118118	8881188188
8181818881	8111181188	1888881818	1181111811	8118818888	8888181111	1118888881	8888811888	1111188111	1188818181	8818888118
1111881881	8118811111	8111188888	1888881188	1188811888	8888818888	8881181111	1181188811	8888181118	1811818818	8811888111
8881118181	1118881818	8888811118	8111188881	1888888888	1811818118	8888118888	8818881111	1188888188	1818118881	8811181881
1811888118	8818818818	1181181188	8888111188	8111188811	1888888881	1818118181	8881118888	8811881111	1881811111	8188181818
8811181111	1818188111	8811818881	1188888888	8881111811	8111188188	1888888811	1881818818	8818818888	1888111118	8181881188
8811818888	8188181111	1881881881	8811111111	1811888881	8818811888	8111188118	1888888188	1888118881	1111111188	8118888888
1181111118	1188811188	1811811811	1818111888	1881818118	1888118181	1111118818	1111818881	1118811111	1888818888	8118181118
8888188888	8881188881	8818888811	8811888188	8811188118	8188188111	8181881881	8118881818	8888881188	8111881181	8111181111
8888811188	8888111811	8881111888	8818818118	8811818181	8811118818	8188118881	8181811111	8118811118	8118181111	1888881181
1188811811	1811811888	1818118118	1881818181	1888118818	1111118881	1111811111	1118811118	1181111188	8111181118	8111818888
8818881181	1881881118	8888118818	8888888118	1118881881	1188811111	8111888111	8118118118	1111888811	8881188888	1811811188
1818181118	8888818181	8118888188	1111888111	1181118881	8111888118	8118111888	1111188881	8881181181	8888181188	1188811118
8118818118	8181888811	1111188118	1118818818	8111888188	8118111811	1888188888	8888181181	1811881118	8888881818	1181111111
8188188881	1888188188	1111818181	8111888811	8118111188	1881881188	8888881181	1188881118	8181811888	8118881881	1118818881
1881888811	1111118118	8111888881	8118111118	1818181818	8118881181	1181181118	8188111811	8811188888	8181888111	1111818818
1888111888	8111888888	8118111111	1811881881	8181881181	1118881118	8811111188	8811881188	1818188881	8188188118	1111118181
8111881188	8118118881	1188888111	8188181181	1111881118	8811811118	8818881818	1811888888	1881811811	8811188188	1888118118
8118118818	1181188118	8811181181	1111181118	8818811111	8881181881	1188881188	1818111188	8111881818	8811888811	1881811888
1118888188	8811881181	1888181118	8881118881	8888188111	1181181818	1811811118	8111881881	8118118181	8818888881	1818111811
116930769296	116930769296	116930769296	116930769296	116930769296	116930769296	116930769296	116930769296	116930769296	116930769296	116930769296
1	2	3	4	5	6	7	8	9	10	11

											116930769296
111888888	118111188	811818818	811181818	181188118	888818188	8881118818	1188818881	8118881118	1881881181	8818811188	116930769296
118118881	1188818118	8111818818	181818888	8881181818	8818818881	1811811111	8888881118	1888181181	8811811811	8118188118	116930769296
1188888811	1811818181	1881888881	8818881188	8811811111	1818111118	8888181118	1111181181	8811111888	8118188111	8111818881	116930769296
1811888188	1818118818	8811888888	8811111118	1881811188	8881181118	1111881181	8188118118	8118181881	8111811111	1888188811	116930769296
1818188118	1881818881	8188111188	1888111811	8818881118	1118881181	8181818181	8118181818	8111811118	1111188188	8811188881	116930769296
1881888111	1888111111	1111111888	8811881118	1181181181	8118818818	8118181188	8111811188	1111888118	8188188811	8181811811	116930769296
1888181881	1111111118	8811181118	1188881181	8888818881	8118188888	8111811811	1118888111	8181888188	8118811888	1111818118	116930769296
1111181818	1111811188	1811881181	8888111111	8118188881	8111811888	1181181881	8118888118	8888818118	1118818181	8188181118	116930769296
1111881188	1118811811	8881111118	8118188811	8111818118	1188881818	8888888111	8888118181	1181118818	8181881118	1818181181	116930769296
8111181181	8111881111	8888888811	8118888881	8181888888	8188181188	8811181818	8811881881	8818888111	8881188118	8888188188	116930769296
8118118888	1888881118	1118818118	1111811888	1111111811	1888111188	1881811118	1818111111	1811818881	1188818818	1181118181	116930769296
8111881118	8111118888	1111881818	1111181881	1888188111	1881888118	1818188188	1811888811	1188888881	1181188888	1118881188	116930769296
1888881111	8118181181	8181888118	8188188188	8811188811	8811888881	8818888888	8881181188	8888181818	8888881881	8118888111	116930769296
8181811118	8811881818	8118881188	1888818881	8111111888	1181188188	1888188881	8811111811	8188118181	1818181111	1111118888	116930769296
8188111111	8818881881	1888118888	8181881818	1888818818	8111111811	1118888811	1881888888	8811811188	8811118118	1811881111	116930769296
8811118881	8881188111	1188881111	1881818888	8188181881	1888818181	8111111188	1111888881	1818181188	8818811118	8811811888	116930769296
8811818818	8888188118	8818811811	1181181111	1818118888	8811188111	1888818118	8111111118	1111188888	1811881818	8881111111	116930769296
8818818181	8888888188	8888118881	8881111188	1118881111	1811818888	8811888118	1888811888	8111111111	1888181188	1188881881	116930769296
8881118118	8118888811	1181188111	8888818818	8888111118	1111881111	1188818888	8818888188	1888811811	8111118881	1881881818	116930769296
8888111888	8181888881	1818181881	1118888118	8118818181	8888811111	1111181111	1181118888	8881188811	1888811188	8111118818	116930769296
8888811811	8188188888	8111118181	1811888111	1111888188	8181818118	8118818881	1888181111	1118818888	8888188881	1888811118	116930769296
8118811188	8811181188	1888811111	8111118118	1188888118	1111188811	8188111888	8181818818	1881881111	1111818888	8888888888	116930769296
116930769296	116930769296	116930769296	116930769296	116930769296	116930769296	116930769296	116930769296	116930769296	116930769296	116930769296	116930769296
12	13	14	15	16	17	18	19	20	21	22	

It is based on 2 by 2 combinations of 22 numbers:

$$\{11111, 11118, 11188, 11811, 11888, 18118, 18181, 18818, 18881, 18888, 81111, 81118, 81181, 81188, 81818, 81881, 88111, 88118, 88188, 88811, 88881, 88888\}.$$

These 22 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{11188, 88111\}$ ,  $\{11888, 88811\}$ ,  $\{18118, 81181\}$ ,  $\{18818, 81881\}$ ,  $\{81188, 88118\}$  and  $\{18888, 88881\}$ . The numbers  $\{11111, 11811, 18181, 81118, 18881, 81818, 88188, 88888\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.9 is **universal magic square**. Its **magic sum** is

$$S_{22 \times 22} := 116930769296 = 100000 \times 1169296 + 1169296,$$

where



$$1169296 := 11111 + 11118 + 11188 + 11811 + 11888 + 18118 + 18181 + 18818 + 18881 + 18888 + 81111 +$$
$$+ 81118 + 81181 + 81188 + 81818 + 81881 + 88111 + 88118 + 88188 + 88811 + 88881 + 88888.$$

2.7 Magic Square of Order 23

**Example 2.10.** *The magic square of order 23 for the digits 1 and 8 is given below in two parts:*

23x23	pan	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877
	1111811118	1118188881	1118888811	1181188118	1188181888	1188881188	1811181111	1818118881	1881118181	1888111888	1888811811	811111181
11888888877	8888111181	8888888888	1111888818	1118188188	1118888111	1181181818	1188181118	1188818888	1811118811	1818118111	1881111881	1888111188
11888888877	8881111188	8881811118	8888188881	8888888811	1111888118	1118181888	1118881188	1181181111	1188118881	1188818181	1811111888	1818111811
11888888877	8811811811	8818811181	8881188888	8881888818	8888188188	8888881111	1111881818	1118181118	1118818888	1181118811	1188118111	1188811881
11888888877	8188811881	8811111188	8811811118	8818888881	8881188811	8881888118	8888181888	8888881188	1111881111	1118118881	1118818181	1181111888
11888888877	8118811888	8181811811	8188811181	8811188888	8811888818	8818888188	8881188111	8881881818	8888181118	8888818888	1111818811	1118118111
11888888877	8111118111	8111811881	8118811188	8181811118	8188888881	8811188811	8811888118	8818881888	8881181188	8881881111	8888118881	8888818181
11888888877	1888118181	1888811888	8111111811	8111811181	8118888888	8181888818	8188888188	8811188111	8811881818	8818881118	8881118888	8881818811
11888888877	1818118811	1881118111	1888111881	1888811188	8111111118	8111888881	8118888811	8181888118	8188881888	8811181188	8811881111	8818818881
11888888877	1188818881	1811118181	1818111888	1881111811	1888111181	1888888888	8111188818	8111888188	8118888111	8181881818	8188881118	8811118888
11888888877	1181118888	1188118811	1188818111	1811111881	1818111188	1881111118	1888188881	1888888811	8111188118	8111881888	8118881188	8181881111
11888888877	1111818111	1118818881	1181118181	1188111888	1188811811	1811111181	1818188888	1881188818	1888188188	1888888111	8111181818	8111881118
11888888877	8888881118	1111818888	1118118811	1118818111	1181111881	1188111188	1188811118	1811188881	1818188811	1881188118	1888181888	1888881188
11888888877	8881881188	8888181111	8888818881	1111818181	1118111888	1118811811	1181111181	1188188888	1188888818	1811188188	1818188111	1881181818
11888888877	8818881818	8881181118	8881818888	8888118811	8888818111	1111811881	1118111188	1118811118	1181188881	1188188811	1188888118	1811181888
11888888877	8811181888	8811881188	8818881111	8881118881	8881818181	8888111888	8888811811	1111811181	1118188888	1118888818	1181188188	1188188111
11888888877	8181888111	8188881818	8811181118	8811818888	8818818811	8881118111	8881811881	8888111188	8888811118	1111888881	1118188811	1118888118
11888888877	8111888118	8118881888	8181881188	8188881111	8811118881	8811818181	8818811888	8881111811	8881811181	8888188888	8888888818	1111888188
11888888877	1888888188	8111188111	8111881818	8118881118	8181818888	8188818811	8811118111	8811811881	8818811188	8881111118	8881888881	8888188811
11888888877	1881188811	1888188118	1888881888	8111181188	8111881111	8118818881	8181818181	8188811888	8811111811	8811811181	8818888888	8881188818
11888888877	1811188818	1818188188	1881188111	1888181818	1888881118	8111118888	8111818811	8118818111	8181811881	8188811188	8811111118	8811888881
11888888877	1188188881	1188888811	1811188118	1818181888	1881181188	1888181111	1888818881	8111118181	8111811888	8118811811	8181811181	8188888888
11888888877	1118888888	1181188818	1188188188	1188888111	1811181818	1818181118	1881118888	1888118811	1888818111	8111111881	8111811188	8118811118
	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877
	1	2	3	4	5	6	7	8	9	10	11	12



11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877
811188888	8118888818	8181888188	8188888111	8811181818	8811881118	8818818888	8881118811	8881818111	8888111881	8888811188	11888888877
1888811118	8111188881	8111888811	8118888118	8181881888	8188881188	8811181111	8811818881	8818818181	8881111888	8881811811	11888888877
1881111181	1888188888	1888888818	8111188188	8111888111	8118881818	8181881118	8188818888	8811118811	8811818111	8818811881	11888888877
1811111188	1818111118	1881188881	1888188811	1888888118	8111181888	8111881188	8118881111	8181818881	8188818181	8811111888	11888888877
1188111811	1188811181	1811188888	1818188818	1881188188	1888188111	1888881818	8111181118	8111818888	8118818811	8181818111	11888888877
1118811881	1181111188	1188111118	1188888881	1811188811	1818188118	1881181888	1888181188	1888881111	8111118881	8111818181	11888888877
1111811888	1118111811	1118811181	1181188888	1188188818	1188888188	1811188111	1818181818	1881181118	1888118888	1888818811	11888888877
8888118111	8888811881	1111811188	1118111118	1118888881	1181188811	1188188118	1188881888	1811181188	1818181111	1881118881	11888888877
8881118181	8881811888	8888111811	8888811181	1111888888	1118188818	1118888188	1181188111	1188181818	1188881118	1811118888	11888888877
8811818811	8818818111	8881111881	8881811188	8888111118	8888888881	1111888811	1118188118	1118881888	1181181188	1188181111	11888888877
8188818881	8811118181	8811811888	8818811811	8881111181	8881888888	8888188818	8888888188	1111888111	1118181818	1118881118	11888888877
8118818888	8181818811	8188818111	8811111881	8811811188	8818811118	8881188881	8881888811	8888188118	8888881888	1111881188	11888888877
8111181111	8111818881	8118818181	8181811888	8188811811	8811111181	8811888888	8818888818	8881188188	8881888111	8888181818	11888888877
1888181118	1888818888	8111118811	8111818111	8118811881	8181811188	8188811118	8811188881	8811888811	8818888118	8881181888	11888888877
1818181188	1881181111	1888118881	1888818181	8111111888	8111811811	8118811181	8181888888	8188888818	8811188188	8811888111	11888888877
1188881818	1811181118	1818118888	1881118811	1888181111	1888811881	8111111188	8111811118	8118888881	8181888811	8188888118	11888888877
1181181888	1188181188	1188881111	1811118881	1818118181	1881111888	1888111811	1888811181	8111188888	8111888818	8118888188	11888888877
1118188111	1118881818	1181181118	1188118888	1188818811	1811118111	1818111881	1881111188	1888111118	1888888881	8111188811	11888888877
8888888118	1111881888	1118181188	1118881111	1181118881	1188118181	1188811888	1811111811	1818111181	1881188888	1888188818	11888888877
8881888188	8888188111	8888881818	1111881118	1118118888	1118818811	1181118111	1188111881	1188811188	1811111118	1818188881	11888888877
8818888811	8881188118	8881881888	8888181188	8888881111	1111818881	1118118181	1118811888	1181111811	1188111181	1188888888	11888888877
8811188818	8811888188	8818888111	8881181818	8881881118	8888118888	8888818811	1111818111	1118111881	1118811188	1181111118	11888888877
8181888881	8188888811	8811188118	8811881888	8818881188	8881181111	8881818881	8888118181	8888811888	1111811811	1118111181	11888888877
11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877	11888888877
13	14	15	16	17	18	19	20	21	22	23	

It is based on 2 by 2 combinations of 23 numbers:

$$\{11118, 11181, 11188, 11811, 11881, 11888, 18111, 18181, 18811, 18881, 18888, 81111, 81118, 81188, 81818, 81888, 88111, 88118, 88188, 88811, 88818, 88881, 88888\}.$$

These 23 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{11188, 88111\}$ ,  $\{11888, 88811\}$ ,  $\{11181, 18111\}$ ,  $\{11881, 18811\}$ ,  $\{81188, 88118\}$ ,  $\{81888, 88818\}$  and  $\{18888, 88881\}$ . The numbers  $\{11811, 18181, 18881, 81118, 81818, 88188, 88888\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.10 is **universal magic square**. Its magic sum is

$$S_{23 \times 23} := 118888888877 = 100000 \times 1188877 + 1188877,$$



where

$1188877 := 11118 + 11181 + 11188 + 11811 + 11881 + 11888 + 18111 + 18181 + 18811 + 18881 + 18888 + 81111 + 81118 + 81188 + 81818 + 81888 + 88111 + 88118 + 88188 + 88811 + 88818 + 88881 + 88888 \}$ .

2.8 Magic Square of Order 24

2.8.1 Blocks of Order 6

**Example 2.11.** *The magic square of order 24 for the digits 1 and 8 is given below in two parts:*

24x24	<u>mgc</u>										
181111111	8188888118	8188881188	8188818811	181111881	1811188888	111111118	8888888188	8888881111	8888818888	111111811	1111188881
8188188888	1811811881	8188181188	1811818811	1811888118	8188111111	8811888881	1188111811	8811881111	1188118888	1188188188	8811811118
8118188888	8118188118	1881818811	1881881188	811811881	1881811111	8118888881	8118888188	1881118888	1881811111	8118811811	1881111118
1881888888	1881811881	8118118811	8118181188	1881888118	8118111111	1881188881	1881111811	8118818888	8118881111	1881188188	8118811118
1811811111	8188111881	1811881188	8188118811	8188188118	1811888888	118811118	8811811811	1188181111	8811818888	8811888188	1188188881
8188811111	1811188118	1811118811	1811181188	8188811881	8188888888	8888811118	1111188188	1111118888	1111181111	8888811811	8888888881
1118118111	8881881881	8881881181	8881818818	1118118118	1118181888	1111811181	8888188811	8888188111	8888111888	1111811188	1111888818
8881181888	1118818118	8881181181	1118818818	1118881881	8881118111	8818888818	1181111188	8818888111	1181111888	1181188811	8818811181
8811181888	8811181881	1188818818	1188881181	8811118118	1188818111	8111188818	8111188811	1888811888	1888888111	8111111888	1888811181
1188881888	1188818118	8811118818	8811181181	1188881881	8811111811	1888888818	1888811188	8111111888	8111188111	1888888811	8111111181
1118818111	8881118118	1118881181	8881118818	8881181881	1118881888	1181111181	8818811188	1181188111	8818811888	8818888811	1181188818
8881818111	1118181881	1118118818	1118181181	8881818118	8881881888	8888111181	1111888811	1111811888	1111888111	8888111188	8888188818
1111811118	8888188188	8888181111	8888118888	1111811811	1111888881	1118111111	8881888118	8881881188	8881818811	1118111881	1118188888
8818888881	1181111811	8818881111	1181118888	1181188188	8818811118	8881188888	1118811881	8881181188	1118818811	1118888118	8881111111
8111188881	8111188188	1888818888	1888881111	8111111811	1888811118	8811188888	8811188118	1188818811	1188881188	8811111881	1188811111
1888888881	1888811811	8111118888	8111181111	1888888188	8111111118	1188888888	1188811881	8811118811	8811181188	1188888118	8811111111
1181111118	8818811811	1181181111	8818818888	8818888188	1181188881	1118811111	8881111881	1118881188	8881118811	8881188118	1118888888
8888111118	1111888188	1111818888	1111881111	8888111811	8888188881	8881811111	1118188118	1118118811	1118181188	8881811881	8881888888
1111111181	8888888811	8888888111	8888811888	1111111188	1111188818	1811118111	8188881881	8188881181	8188818818	1811118118	1811181888
8811888818	1188111188	8811888111	1188111888	1188188811	8811811181	8188181888	1811818118	8188181181	1811818818	1811881881	8188118111
8118888818	8118888811	1881111888	1881188111	8118811188	1881111181	8118181888	8118181881	1881818818	1881881181	8118118118	1881818111
1881188818	1881111188	8118811888	8118888111	1881188811	8118811181	1881881888	1881818118	8118118818	8118181181	1881881881	8118118111
1188111181	8811811188	1188188111	8811811888	8811888811	1188188818	1811818111	8188118118	1811881181	8188118818	8188181881	1811881888
8888811181	1111188811	1111111888	1111188111	8888811188	8888888818	8188818111	1811181881	1811118818	1811181181	8188818118	8188881888
11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988
1	2	3	4	5	6	7	8	9	10	11	12

												11999999988
111811181	888188811	888188811	888181188	111811188	1118188818	111181811	8888181881	8888181181	8888118818	1111818118	1111881888	11999999988
8881188818	1118811188	8881188111	1118811888	1118888811	8881111181	8818881888	1181118118	8818881181	1181118818	1181181881	8818818111	11999999988
8811188818	8811188811	1188811888	1188888111	8811111188	1188811181	8111181888	8111181881	1888818818	1888881181	811118118	1888818111	11999999988
1188888818	1188811188	8811111888	8811188111	1188888811	8811111181	1888881888	1888818118	811118818	8111181181	1888881881	811118111	11999999988
1118811181	8881111188	1118888111	8881111888	8881188811	1118888818	1181118111	8818818118	1181181181	8818818818	8818881881	1181181888	11999999988
8881811181	1118188811	1118111888	1118188111	8881811188	8881888818	8888118111	1111881881	1111818818	1111881181	8888118118	8888181888	11999999988
1811111118	8188888188	8188881111	8188818888	1811111811	1811188881	1111111111	8888888118	8888881188	8888818811	1111111881	1111188888	11999999988
8188188881	1811811811	8188181111	1811818888	1811888188	8188111118	8811888888	1188111881	8811881188	1188118811	1188188118	8811811111	11999999988
8118188881	8118188188	1881818888	1881881111	8118111811	1881811118	8118888888	8118888118	1881118811	1881181188	8118811881	1881111111	11999999988
1881888881	1881811811	8118118888	8118181111	1881888188	8118111118	1881188888	1881111881	8118818811	8118881188	1881188118	8118811111	11999999988
1811811118	8188111811	1811881111	8188118888	8188188188	1811888881	1188111111	8811811881	1188181188	8811818811	8811888118	1188188888	11999999988
8188811118	1811188188	1811118888	1811181111	8188811811	8188888881	8888811111	1111188118	1111118811	1111181188	8888811881	8888888888	11999999988
1111118111	8888881881	8888881181	8888818818	1111118118	1111181888	1811111181	8188888811	8188888111	8188811888	1811111188	1811188818	11999999988
8811881888	1188118118	8811881181	1188118818	1188181881	8811818111	8188188818	1811811188	8188188111	1811811888	1811888811	8188111181	11999999988
8118881888	8118881881	1881118818	1881181181	8118818118	1881118111	8118188818	8118188811	1881811888	1881888111	8118111188	1881811181	11999999988
1881181888	1881118118	8118818818	8118881181	1881181881	8118818111	1881888818	1881811188	8118111888	8118188111	1881888811	8118111181	11999999988
1188118111	8811818118	1188181181	8811818818	8811881881	1188181888	1811811181	8188111188	1811888111	8188111888	8188188811	1811888818	11999999988
8888818111	1111181881	1111118818	1111181181	8888818118	8888881888	8188811181	1811188811	1811111888	1811188111	8188811188	8188888818	11999999988
1111811111	8888188118	8888181188	8888118811	1111811881	1111888888	1118111118	8881888188	8881881111	8881818888	1118111811	1118188881	11999999988
8818888888	1181111881	8818881188	1181118811	1181188118	8818811111	8881188881	1118811811	8881181111	1118818888	1118888188	8881111118	11999999988
8111188888	8111188118	1888818811	1888881188	8111111881	1888811111	8811188881	8811188188	1188818888	1188881111	8811111811	1188811118	11999999988
1888888888	1888811881	8111118811	8111181188	1888888118	8111111111	1188888881	1188811811	8811118888	8811181111	1188888188	8811111118	11999999988
1181111111	8818811881	1181181188	8818818811	8818888118	1181188888	1118811118	8881111811	1118881111	8881118888	8881188188	1118888881	11999999988
8888111111	1111888118	1111818811	1111881188	8888111881	8888188888	8881811118	1118188188	1118118888	1118181111	8881811811	8881888881	11999999988
11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988
13	14	15	16	17	18	19	20	21	22	23	24	

It is based on 2 by 2 combinations of 24 numbers:

$$\{11111, 11181, 11811, 11818, 11881, 18111, 18118, 18181, 18188, 18811, 18818, 18881, 81118, 81181, 81188, 81811, 81818, 81881, 81888, 88118, 88181, 88188, 88818, 88888\}.$$

These 24 numbers are **upside-down** and **mirror looking** in pairs:  $\{11181, 18111\}$ ,  $\{11818, 81811\}$ ,  $\{11881, 18811\}$ ,  $\{18118, 81181\}$ ,  $\{18188, 88181\}$ ,  $\{18818, 81881\}$ ,  $\{81188, 88118\}$  and  $\{81888, 88818\}$ . The numbers  $\{11111, 11811, 18181, 18881, 81118, 81818, 88188, 88888\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.11 is **universal magic square**. Its **magic sum** is



$$S_{24 \times 24} := 119999999988 = 100000 \times 1199988 + 1199988,$$

where

$$\begin{aligned} 1199988 := & 11111 + 11181 + 11811 + 11818 + 11881 + 18111 + 18118 + 18181 + \\ & + 18188 + 18811 + 18818 + 18881 + 81118 + 81181 + 81188 + 81811 + \\ & + 81818 + 81881 + 81888 + 88118 + 88181 + 88188 + 88818 + 88888. \end{aligned}$$

The blocks of order 6 appearing in Example 2.11 are of equal magic sums. The blocks are constructed dividing the above 24 numbers in four parts with equal sums:

$$\begin{aligned} A &:= 11181 + 11188 + 11888 + 88111 + 88811 + 88818 = 299997 \\ B &:= 18111 + 18118 + 18818 + 81181 + 81881 + 81888 = 299997 \\ C &:= 11111 + 11881 + 18811 + 81188 + 88118 + 88888 = 299997 \\ D &:= 11118 + 11811 + 18888 + 81111 + 88188 + 88881 = 299997 \end{aligned}$$

Based on these four groups  $A, B, C$  and  $D$ , 16 magic squares of order 6 are constructed with equal magic sums  $S_{6 \times 6} := 29999999997$ . These 16 magic squares lead us to a magic square of order 24 given in Example 2.11.

## 2.8.2 Blocks of Order 4

**Example 2.12.** *The pandiagonal magic square of order 24 for the digits 1 and 8 is given below in two parts:*

24x24	pan	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988
	1811181888	8188888818	1118111181	8881818111	1811181811	8188888181	1118111818	8881818188	1811181188	8188888118	1118111881	8881818811
11999999988	1118118111	8881811181	1811188818	8188881888	1118118188	8881811818	1811188181	8188881811	1118118811	8881811881	1811188118	8188881188
11999999988	8881888818	1118181888	8188818111	1811111181	8881888181	1118181811	8188818188	1811111818	8881888118	1118181188	8188818811	1811111881
11999999988	8188811181	1811118111	8881881888	1118188818	8188811818	1811118188	8881881811	1118188181	8188811881	1811118811	8881881188	1118188118
11999999988	1818881888	8181188818	1181811181	8818118111	1818881811	8181188181	1181811818	8818118188	1818881188	8181188118	1181811881	8818118811
11999999988	1181818111	8818111181	1818888818	8181181888	1181818188	8818111818	1818888181	8181181811	1181818811	8818111881	1818888118	8181181188
11999999988	8818188818	1181881888	8181118111	1818811181	8818188181	1181881811	8181118188	1818811818	8818188118	1181881188	8181118811	1818811881
11999999988	8181111181	1818818111	8818181888	1181888818	8181111818	1818818188	8818181811	1181888181	8181111881	1818818811	8818181188	1181888118
11999999988	1881181888	8118888818	1188111181	8811818111	1881181811	8118888181	1188111818	8811818188	1881181188	8118888118	1188111881	8811818811
11999999988	1188118111	8811811181	1881188818	8118881888	1188118188	8811811818	1881188181	8118881811	1188118811	8811811881	1881188118	8118881188
11999999988	8811888818	1188181888	8118818111	1881111181	8811888181	1188181811	8118818188	1881111818	8811888118	1188181188	8118818811	1881111881
11999999988	8118811181	1881118111	8811881888	1188188818	8118811818	1881118188	8811881811	1188188181	8118811881	1881118811	8811881188	1188188118
11999999988	1881881888	8118188818	1811811181	8188118111	1881881811	8118188181	1811811818	8188118188	1881881188	8118188118	1811811881	8188118811
11999999988	1811818111	8188111181	1881888818	8118181888	1811818188	8188111818	1881888181	8118181811	1811818811	8188111881	1881888118	8118181188
11999999988	8188188818	1811881888	8118118111	1881811181	8188188181	1811881811	8118118188	1881811818	8188188118	1811881188	8118118811	1881811881
11999999988	8118111181	1881818111	8188181888	1811888818	8118111818	1881818188	8188181811	1811888181	8118111881	1881818811	8188181188	1811888118
11999999988	1888181888	8111888818	1818111181	8181818111	1888181811	8111888181	1818111818	8181818188	1888181188	8111888118	1818111881	8181818811
11999999988	1818118111	8181811181	1888188818	8111881888	1818118188	8181811818	1888188181	8111881811	1818118811	8181811881	1888188118	8111881188
11999999988	8181888818	1818181888	8111818111	1888111181	8181888181	1818181811	8111818188	1888111818	8181888118	1818181188	8111818811	1888111881
11999999988	8111811181	1888118111	8181881888	1818188818	8111811818	1888118188	8181881811	1818188181	8111811881	1888118811	8181881188	1818188118
11999999988	1181181888	8818888818	1111111181	8888818111	1181181811	8818888181	1111111818	8888818188	1181181188	8818888118	1111111881	8888818811
11999999988	1111118111	8888811181	1181188818	8818881888	1111118188	8888811818	1181188181	8818881811	1111118811	8888811881	1181188118	8818881188
11999999988	8888888818	1111181888	8818818111	1181111181	8888888181	1111181811	8818818188	1181111818	8888888118	1111181188	8818818811	1181111881
11999999988	8818811181	1181118111	8888881888	1111188818	8818811818	1181118188	8888881811	1111188181	8818811881	1181118811	8888881188	1111188118
	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988
	1	2	3	4	5	6	7	8	9	10	11	12



119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988
1811181181	8188881881	1118118118	8881818818	1811181118	8188881818	1118118181	8881818881	1811188188	8188888888	1118111111	8881811811	119999999988
1118118818	8881818118	1811181881	8188881181	1118118881	8881818181	1811181818	8188881118	1118111811	8881811111	1811188888	8188888188	119999999988
8881881881	1118181181	8188818818	1811118118	8881881818	1118181118	8188818881	1811118181	8881888888	1118188188	8188811811	1811111111	119999999988
8188818118	1811118818	8881881181	1118181881	8188818181	1811118881	8881881118	1118181818	8188811111	1811111811	8881888188	1118188888	119999999988
1818881181	8181181881	1181818118	8818118818	1818881118	8181181818	1181818181	8818118881	1818888188	8181188888	1181811111	8818111811	119999999988
1181818818	8818118118	1818881881	8181181181	1181818881	8818118181	1818881818	8181181118	1181811811	8818111111	1818888888	8181188188	119999999988
8818181881	1181881181	8181118818	1818818118	8818181818	1181881118	8181118881	1818818181	8818188888	1181888188	8181111811	1818811111	119999999988
8181118118	1818818818	8818181181	1181881881	8181118181	1818818881	8818181118	1181881818	8181111111	1818811811	8818188188	1181888888	119999999988
1881811818	8118881881	1188118118	8811818818	1881811118	8118881818	1188118181	8811818881	1881188188	8118888888	1188111111	8811811811	119999999988
1188118818	8811818118	1881181881	8118881181	1188118881	8811818181	1881181818	8118881118	1188111811	8811811111	1881188888	8118888188	119999999988
8811881881	1188181181	8118818818	1881118118	8811881818	1188181118	8118818881	1881118181	8811888888	1188188188	8118811811	1881111111	119999999988
8118818118	1881118818	8811881181	1188181881	8118818181	1881118881	8811881118	1188181818	8118811111	1881111811	8811888188	1188188888	119999999988
1881881181	8118181881	1811818118	8188118818	1881881118	8118181818	1811818181	8188118881	1881888188	8118188888	1811811111	8188111811	119999999988
1811818818	8188118118	1881881881	8118181181	1811818881	8188118181	1881881818	8118181118	1811811811	8188111111	1881888888	8118188188	119999999988
8188181881	1811881181	8118118818	1881818118	8188181818	1811881118	8118118881	1881818181	8188188888	1811888188	8118111811	1881811111	119999999988
8118118118	1881818818	8188181181	1811881881	8118118181	1881818881	8188181118	1811881818	8118111111	1881811811	8188188188	1811888888	119999999988
1888181181	8111881881	1818118118	8181818818	1888181118	8111881818	1818118181	8181818881	1888188188	8111888888	1818111111	8181811811	119999999988
1818118818	8181818118	1888181881	8111881181	1818118881	8181818181	1888181818	8111881118	1818111811	8181811111	1888188888	8111888188	119999999988
8181881881	1818181181	8111818818	1888118118	8181881818	1818181118	8111818881	1888118181	8181888888	1818188188	8111811811	1888111111	119999999988
8111818118	1888118818	8181881181	1818181881	8111818181	1888118881	8181881118	1818181818	8111811111	1888111811	8181888188	1818188888	119999999988
1181181181	8818881881	1111181118	8888818818	1181181118	8818881818	1111118181	8888818881	1181188188	8818888888	1111111111	8888811811	119999999988
1111118818	8888818118	1181181881	8818881181	1111118881	8888818181	1181181818	8818881118	1111111811	8888811111	1181188888	8818888188	119999999988
8888881881	1111181181	8818818818	1181118118	8888881818	1111181118	8818818881	1181118181	8888888888	1111188188	8818811811	1181111111	119999999988
8818818118	1181118818	8888881181	1111181881	8818818181	1181118881	8888881118	1111181818	8818811111	1181111811	8888888188	1111188888	119999999988
119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988	119999999988
13	14	15	16	17	18	19	20	21	22	23	24	

This magic square is also constructed with the same entries as of Example 2.11. As explained above these numbers lead us to a **universal magic square** with same magic sum, i.e.,

$$S_{24 \times 24} := 119999999988 = 100000 \times 1199988 + 1199988,$$

The magic square given in Example 2.12 is constructed based on blocks of order 4. These blocks are **pan-diagonal** with equal magic sums resulting in a **pandiagonal magic square** of order 24. The blocks are constructed dividing the 24 numbers given in Example 2.11 in six groups of equal sums:

$$A := 11181 + 18111 + 81888 + 88818 = 199998$$

$$B := 11818 + 18188 + 81811 + 88181 = 199998$$

$$C := 11881 + 18811 + 81188 + 88118 = 199998$$

$$D := 18118 + 18818 + 81181 + 81881 = 199998$$

$$E := 18181 + 18881 + 81118 + 81818 = 199998$$

$$F := 11111 + 11811 + 88188 + 88888 = 199998$$

Based on these six groups  $A, B, C, D, E$  and  $F$ , we get 24 **pandiagonal magic squares** of order 4 with equal **magic sums**:  $S_{4 \times 4} := 1999999998$ .. These 36 **pandiagonal magic squares** of order 4 lead us to a **pandiagonal magic square** of order 24 as given in Example 2.12.

### 2.8.3 Blocks of Order 12

**Example 2.13.** *The pandiagonal magic square of order 24 for the digits 1 and 8 is given below in two parts:*



24x24	mgc										
8881118111	8188118818	8881811888	1188881888	1881881181	1811188111	1118811181	1811818118	1118111188	8811188818	8118181881	8188888811
8881818818	8881111888	8188118111	1811181181	1188888111	1881881888	1118118118	1118811188	1811811181	8188881881	8811188811	8118188818
8188111888	8881818111	8881118818	1881888111	1811181888	1188881181	1811811188	1118111181	1118818118	8118188811	8188888818	8811181881
1118888818	1811881881	1118188811	8811111181	8118118118	8188811188	8881181888	8188181181	8881888111	1188818111	1881818818	1811111888
1118181881	1118888811	1811888818	8188818118	8811111188	8118111181	8881881181	8881188111	8188181888	1811118818	1188811888	1881818111
1811888811	1118188818	1118881881	8118111188	8188811181	8811118118	8188188111	8881881888	8881181181	1881811888	1811118111	1188818818
8811181888	8118181181	8188888111	1118818111	1811818818	1118111888	1188888818	1881881881	1811188811	8881111181	8188118118	8881811188
8188881181	8811188111	8118181888	1118118818	1118811888	1811818111	1811181881	1188888811	1881888818	8881818118	8881111188	8188111181
8118188111	8188881888	8811181181	1811811888	1118118111	1118818818	1881888811	1811188818	1188881881	8188111188	8881811181	8881118118
1188811181	1881818118	1811111188	8881188818	8188181881	8881888811	8811118111	8118118818	8188811888	1118881888	1811881181	1118188111
1811118118	1188811188	1881811181	8881881881	8881188811	8188188818	8188818818	8811111888	8118118111	1118181181	1118888111	1811881888
1881811188	1811111181	1188818118	8188188811	8881888818	8881181881	8118111888	8188818111	8811118818	1811888111	1118181888	1118881181
8111118111	1881118818	8888811888	1888881888	8118881181	1181188111	1111811181	1188118118	1111111188	8888188818	8811881881	8818888811
8888818818	8111111888	1881118111	1181181181	1888888111	8118881888	1111118118	1111811188	1188111181	8818881881	8888188811	8811888818
1881111888	8888818111	8111118818	8118888111	1181181888	1888881181	1188111188	1111111181	1111818118	8811888811	8818888818	8888181881
1111888818	1188181881	1111188811	8888111181	8811818118	8818811188	8111181888	1881181181	8888888111	1888818111	8118818818	1181111888
1111181881	1111888811	1188188818	8818818118	8888111188	8811811181	8888881181	8111188111	1881181888	1181118818	1888811888	8118818111
1188188811	1111188818	1111881881	8811811188	8818811181	8888118118	1881188111	8888881888	8111181181	8118811888	1181118111	1888818818
8888181888	8811881181	8818888111	1111818111	1188118818	1111111888	1888888818	8118881881	1181188811	8111111181	1881118118	8888811188
8818881181	8888188111	8811881888	1111118818	1111811888	1188118111	1181181881	1888888811	8118888818	8888818118	8111111188	1881111181
8811888111	8818881888	8888181181	1188111888	1111118111	1111818818	8118888811	1181188818	1888881881	1881111188	8888811181	8111118118
1888811181	8118818118	1181111188	8111188818	1881181881	8888888811	8888118111	8811818818	8818811888	1111881888	1188181181	1111188111
1181118118	1888811188	8118811181	8888881881	8111188811	1881188818	8818818818	8888111888	8811818111	1111181181	1111888111	1188181888
8118811188	1181111181	1888818118	1881188811	8888888818	8111181881	8811811888	8818818111	8888118818	1188188111	1111181888	1111881181
11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988
1	2	3	4	5	6	7	8	9	10	11	12

												11999999988
888111811	818818188	888181888	1188888188	1881888118	1811188881	1118811111	1811811881	1118111118	8811188888	8118118811	8188881111	11999999988
8881881188	8881118888	8188111811	1811188118	1188888881	1881888188	1118111881	1118811118	1811811111	8188818811	8811181111	8118188888	11999999988
8188118888	8881811811	8881181188	1881888881	1811188188	1188888118	1811811118	1118111111	1118811881	8118181111	8188888888	8811118811	11999999988
1118888888	1811818811	1118181111	8811111111	8118111881	8188811118	8881188188	8188188118	8881888881	1188811811	1881881188	1811118888	11999999988
1118118811	1118881111	1811888888	8188811881	8811111118	8118111111	8881888118	8881188881	8188188188	1811181188	1188818888	1881811811	11999999988
1811881111	1118188888	1118818811	8118111118	8188811111	8811111881	8188188881	8881888188	8881188118	1881818888	1811111811	1188881188	11999999988
8811188188	8118188118	8188888881	1118811811	1811881188	1118118888	1188888888	1881818811	1811181111	8881111111	8188111881	8881811118	11999999988
8188888118	8811188881	8118188188	1118181188	1118818888	1811811811	1811118811	1188881111	1881888888	8881811881	8881111118	8188111111	11999999988
8118188881	8188888188	8811188118	1811818888	1118111811	1118881188	1881881111	1811188888	1188818811	8188111118	8881811111	8881111881	11999999988
1188811111	1881811881	1811111118	8881188888	8188118811	8881881111	8811111811	8118181188	8188818888	1118888188	1811888118	1118188881	11999999988
1811118811	1188811118	1881811111	8881818811	8881181111	8188188888	8188881188	8811118888	8118111811	1118188118	1118888881	1811888188	11999999988
1881811118	1811111111	1188811881	8188181111	8881888888	8881118811	8118118888	8188811811	8811181188	1811888881	1118188188	1118888118	11999999988
8111111811	1881181188	8888818888	1888888188	8118888118	1181188881	1111811111	1188111881	1111111118	8888188888	8811818811	8818881111	11999999988
8888881188	8111118888	1881111811	1181188118	1888888881	8118888188	1111111881	1111811118	1188111111	8818818811	8888181111	8811888888	11999999988
1881118888	8888811811	8111181188	8118888881	1181188188	1888888118	1188111118	1111111111	1111811881	8811881111	8818888888	8888118811	11999999988
1111888888	1188118811	1111181111	8888111111	8811811881	8818811118	8111188188	1881188118	8888888881	1888811811	8118881188	1181118888	11999999988
1111118811	1111881111	1188188888	8818811881	8888111118	8811811111	8888888118	8111188881	1881188188	1181181188	1888818888	8118811811	11999999988
1188181111	1111188888	1111818811	8811811118	8818811111	8888111881	1881188881	8888888188	8111188118	8118818888	1181111811	1888881188	11999999988
8888188188	8811888118	8818888881	1111811811	1188181188	1111118888	1888888888	8118818811	1181181111	8111111111	1881111881	8888811118	11999999988
8818888118	8888188881	8811888188	1111181188	1111818888	1188111811	1181118811	1888881111	8118888888	8888811881	8111111118	1881111111	11999999988
8811888881	8818888188	8888188118	1188118888	1111111811	1111881188	8118881111	1181188888	1888818811	1881111118	8888811111	8111111881	11999999988
1888811111	8118811881	1181111118	8111188888	1881118811	8888881111	8888111811	8811881188	8818818888	1111888188	1188188118	1111188881	11999999988
1181111881	1888811118	8118811111	8888818811	8111181111	1881188888	8818881188	8888118888	8811811811	1111188118	1111888881	1188188188	11999999988
8118811118	1181111111	1888811881	1881181111	8888888888	8111118811	8811818888	8818811811	8888181188	1188188881	1111188188	1111888118	11999999988
11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988	11999999988
13	14	15	16	17	18	19	20	21	22	23	24	

This magic square is also constructed with the same number as of Example 2.11. As explained above these numbers lead us to a **universal magic square** with same magic sum, i.e.,

$$S_{24 \times 24} := 119999999988 = 100000 \times 1199988 + 1199988,$$

The magic square given in Example 2.13 is constructed based on blocks of order 12. The blocks are constructed dividing the 24 numbers given in Example 2.11 in two groups of equal sums:

$$A := 11181 + 11188 + 11888 + 18111 + 18118 + 18818 + 81181 + 81881 + 81888 + 88111 + 88811 + 88818 = 599994$$

$$B := 11111 + 11118 + 11811 + 11881 + 18811 + 18888 + 81111 + 81188 + 88118 + 88188 + 88881 + 88888 = 599994$$



Based on these two groups  $A$  and  $B$  we get 4 **magic squares** of order 12 with equal **magic sums**:  $S_{4 \times 4} := 599999999994$ . These 4 magic square of order 12 lead us to a **magic square** of order 24 as given in Example 2.13. Moreover, each block of order 12 has 16 blocks of order 3. These order 3 blocks are **semi-magic squares** of order 3 with different magic sums.

Finally, we have a magic square of order 24 with 64 blocks of **semi-magic squares** of order 3 with different magic sums and 4 blocks of order 12 with equal magic sums.

#### 2.8.4 Semi-Bimagic Square

**Example 2.14.** *The magic square of order 24 for the digits 1 and 8 is given below in three parts:*

24x24	mgc	semi-bi					
111818888888	888118111111	888111111188	111811888811	111888111181	888188888818	888181888881	111881111118
111888111118	888188888881	888181888818	111881111181	111818888811	888118111188	888111111111	111811888888
111811111111	888111888888	888118888811	111818111188	111881888818	888181111181	888188111118	111888888881
111881888881	888181111118	888188111181	111888888818	111811111188	888111888811	888118888888	111818111111
888111888818	111811111181	111818111118	888118888881	888181111111	111881888888	111888888811	888188111188
888181111188	111881888811	111888888888	888188111111	888111888881	111811111118	111818111181	888118888818
888118111181	111818888818	111811888881	888111111118	888188888888	111888111111	111881111188	888181888811
888188888811	111888111188	111881111111	888181888888	888118111118	111818888881	111811888818	888111111181
111118881888	888818118111	888811118188	111118818111	111188118181	888888881818	888881881881	111181118118
111188118118	888888881881	888881881818	111181118181	111118881811	888818118188	888811118111	11111881888
111111118111	888811881888	888818881811	111118118188	111181881818	888881118181	888888118118	111188881881
111181881881	888881118118	888888118181	111188881818	111111118188	888811881811	888818881888	111118118111
888811881818	111111118181	111118118118	888818881881	888881118111	111181881888	111188881811	888888118188
888881118188	111181881811	111188881888	888888118111	888811881881	111111118118	111118118181	888818881818
888818118181	111118881818	111111881881	888811118118	888888881888	111188118111	111181118188	888881881811
888888881811	111188118188	111181118111	888881881888	888818118118	111118881881	111111881818	888811118181
118118888188	881818111811	881811111888	118111888111	118188111881	881888888118	881881888181	118181111818
118188111818	881888888181	881881888118	118181111881	118118888111	881818111888	881811111811	118111888188
118111111811	881811888188	881818888111	118118111888	118181888118	881881111881	881888111818	118188888181
118181888181	881881111818	881888111881	118188888118	118111111888	881811888111	881818888188	118118111811
881811888118	118111111881	118118111818	881818888181	881881111811	118181888188	118188888111	881888111888
881881111888	118181888111	118188888188	881888111811	881811888181	118111111818	118118111881	881818888118
881818111881	118118888118	118111888181	881811111818	881888888188	118188111811	118181111888	881881888111
881888888111	118188111888	118181111811	881881888188	881818111818	118118888181	118111888118	881811111881
1199999999988	1199999999988	1199999999988	1199999999988	1199999999988	1199999999988	1199999999988	1199999999988
1	2	3	4	5	6	7	8



118118881888	881818118111	881811118188	118111881811	118188118181	881888881818	881881881881	118181118118
118188118118	881888881881	881881881818	118181118181	118118881811	881818118188	881811118111	118111881888
118111118111	881811881888	881818881811	118118118188	118181881818	881881118181	881888118118	118188881881
118181881881	881881118118	881888118181	118188881818	118111118188	881811881811	881818881888	118118118111
881811881818	118111118181	118118118118	881818881881	881881118111	118181881888	118188881811	881888118188
881881118188	118181881811	118188881888	881888118111	881811881881	118111118118	118118118181	881818881818
881818118181	118118881818	118111881881	881811118118	881888881888	118188118111	118181118188	881881881811
881888881811	118188118188	118181118111	881881881888	881818118118	118118881881	118111881818	881811118181
111818888188	888118111811	888111111888	111811888111	111888111881	888188888118	888181888181	111881111818
111888111818	888188888181	888181888118	111881111881	111818888111	888118111888	888111111811	111811888188
111811111811	888111888188	888118888111	111818111888	111881888118	888181111881	888188111818	111888888181
111881888181	888181111818	888188111881	111888888118	111811111888	888111888111	888118888188	111818111811
888111888118	111811111881	111818111818	888118888181	888181111811	111881888188	111888888111	888188111888
888181111888	111881888111	111888888188	888188111811	888111888181	111811111818	111818111881	888118888118
888118111881	111818888118	111811888181	888111111818	888188888188	111888111811	111881111888	888181888111
888188888111	111888111888	111881111811	888181888188	888118111818	111818888181	111811888118	888111111881
111118888888	888818111111	888811111188	111111888811	1111881111181	888888888818	888881888881	111181111118
111188111118	888888888881	888881888818	1111811111181	111118888811	888818111188	888811111111	111111888888
111111111111	888811888888	888818888811	1111181111188	111181888818	888881111181	888888111118	111188888881
111181888881	888881111118	8888881111181	111188888818	1111111111188	888811888811	888818888888	111118111111
888811888818	1111111111181	1111181111118	888818888881	888881111111	111181888888	111188888811	888888111188
888881111188	111181888811	111188888888	888888111111	888811888881	111111111118	1111181111181	888818888818
8888181111181	1111118888818	111111888881	888811111118	888888888888	111188111111	1111811111188	888881888811
888888888811	1111881111188	111181111111	888881888888	888818111118	1111118888881	111111888818	888811111181
1199999999988	1199999999988	1199999999988	1199999999988	1199999999988	1199999999988	1199999999988	1199999999988
9	10	11	12	13	14	15	16

								1199999999988
11118888188	888818111811	88881111888	11111888111	111188111881	888888888118	888881888181	111181111818	1199999999988
111188111818	888888888181	888881888118	111181111881	111118888111	888818111888	888811111811	11111888188	1199999999988
11111111811	888811888188	888818888111	111118111888	111181888118	888881111881	888888111818	111188888181	1199999999988
111181888181	888881111818	888888111881	111188888118	111111111888	888811888111	888818888188	111118111811	1199999999988
888811888118	111111111881	111118111818	888818888181	888881111811	111181888188	111188888111	888888111888	1199999999988
888881111888	111181888111	111188888188	888888111811	888811888181	111111111818	111118111881	888818888118	1199999999988
888818111881	111118888118	111111888181	888811111818	888888888188	111188111811	111181111888	888881888111	1199999999988
888888888111	111188111888	111181111811	888881888188	888818111818	111118888181	111111888118	888811111881	1199999999988
118118888888	881818111111	881811111188	118111888811	118188111181	881888888818	881881888881	118181111118	1199999999988
118188111118	881888888881	881881888818	118181111181	118118888811	881818111188	881811111111	118111888888	1199999999988
118111111111	881811888888	881818888811	118118111188	118181888818	881881111181	881888111118	118188888881	1199999999988
118181888881	881881111118	881888111181	118188888818	118111111188	881811888811	881818888888	118118111111	1199999999988
881811888818	118111111181	118118111118	881818888881	881881111111	118181888888	118188888811	881888111188	1199999999988
881881111188	118181888811	118188888888	881888111111	881811888881	118111111118	118118111181	881818888818	1199999999988
881818111181	118118888818	118111888881	881811111118	881888888888	118188111111	118181111188	881881888811	1199999999988
881888888811	118188111188	118181111111	881881888888	881818111118	118118888881	118111888818	881811111181	1199999999988
111818881888	888118118111	888111118188	111811881811	111888118181	888188881818	888181881881	111881118118	1199999999988
111888118118	888188881881	888181881818	111881118181	111818881811	888118118188	888111118111	111811881888	1199999999988
111811118111	888111881888	888118881811	111818118188	111881881818	888181118181	888188118118	111888881881	1199999999988
111881881881	888181118118	888188118181	111888881818	111811118188	888111881811	888118881888	111818118111	1199999999988
888111881818	111811118181	111818118118	888118881881	888181118111	111811118118	111818118181	888118881818	1199999999988
888181118188	111881881811	111888881888	888188118111	888111881881	111811118118	111818118181	888118881818	1199999999988
888118118181	111818881818	111811881881	888111118118	888188881888	111888118111	111881118188	888181881811	1199999999988
888188881811	111888118188	111881118111	888181881888	888118118118	111818881881	111811881818	888111118181	1199999999988
1199999999988	1199999999988	1199999999988	1199999999988	1199999999988	1199999999988	1199999999988	1199999999988	1199999999988
17	18	19	20	21	22	23	24	

It is based on 2 by 2 combinations of 24 numbers of 6-digits. These are divided in three groups of eight members each with equal sums. See below:

$A := 111111 + 111118 + 111181 + 111188 + 888811 + 888818 + 888881 + 888888 = 3999996$

$B := 111811 + 111818 + 111881 + 111888 + 888111 + 888118 + 888181 + 888188 = 3999996$

$C := 118111 + 118118 + 118181 + 118188 + 881811 + 881818 + 881881 + 881888 = 3999996$

These 24 numbers are chosen in little different way. Symmetrizing compliments of 1 and 8, see them as:  
{111118, 888881}, {111181, 888818}, {111188, 888811}, {111111, 888888}, {111811, 888188}, {111818, 888181},  
{111881, 888118}, {111888, 888111}, {118111, 881888}, {118118, 881881}, {118181, 881818} and {118188, 881811}.



Let's see how they become **upside down** and **mirror looking**:

$$A := 111111 + 811111 + 181111 + 881111 + 118888 + 818888 + 188888 + 888888 = 3999996$$

$$B := 118111 + 818111 + 188111 + 888111 + 111888 + 811888 + 181888 + 881888 = 3999996$$

$$C := 111811 + 811811 + 181811 + 881811 + 118188 + 818188 + 188188 + 888188 = 3999996$$

Eventhough the numbers are different but the sum is same. Thus it is a **universal semi-bimagic square** with **magic sums**:

$$S_{24 \times 24} := 11999999999988 = 1000000 \times 11999988 + 11999988,$$

and

$$S_{b_{24 \times 24}} := 9581393369685581393369700 \quad \text{only rows and columns}$$

where

$$\begin{aligned} 11999988 := & 111111 + 111118 + 111181 + 111188 + 888811 + 888818 + 888881 + 888888 + \\ & 111811 + 111818 + 111881 + 111888 + 888111 + 888118 + 888181 + 888188 + \\ & 118111 + 118118 + 118181 + 118188 + 881811 + 881818 + 881881 + 881888 \end{aligned}$$

Moreover the above 9 magic squares of order 8 are **pandiagonal** either **bimagic** or **semi-bimagic** squares with equal magic sums:

$$S_{8 \times 8} := 3999999999996 = 1000000 \times 3999996 + 3999996,$$

**Remark 2.2.** This *semi-bimagic square* of order 24 is with 12-digits distributions of two numbers 1 and 8. The magic squares given in Examples 2.11, 2.12 and 2.13 are with 10-digits distributions of two numbers 1 and 8.

## 2.9 Magic Square of Order 25

### 2.9.1 Magic Square

**Example 2.15.** The *magic square* of order 25 for the digits 1 and 8 is given below in three parts:

25x25	pan	128888888876	128888888876	128888888876	128888888876	128888888876	128888888876	128888888876
	1111811118	1811188111	8111118111	8811188888	8888881111	8188881888	8888111888	1188188881
128888888876	8811118111	8888888888	1111881111	1811111118	8111188111	1188888881	1888818888	8188811881
128888888876	1811181111	8111111118	8811188111	8888818111	1111888888	8888111881	1188181888	1188811888
128888888876	8888888111	1111818111	1811188888	8111181111	8811111118	1888811888	8188888881	8888118888
128888888876	8111188888	8811181111	8888811118	1111888111	1811118111	1188118888	1188811881	1888881888
128888888876	8881118811	1118888818	1818118818	8118811811	8181881881	8118188811	8811881188	8818811188
128888888876	8118818818	8181811811	8881181881	1118818811	1818188818	1118111188	1811881818	8118118181
128888888876	1118881881	1818118811	8118888818	8181818818	8881111811	8811818181	8818888811	1118181188
128888888876	8181888818	8881118818	1118811811	1818181881	8118818811	1811881188	8118111188	8811881818
128888888876	1818111811	8118881881	8181818811	8881188818	1118818818	8818881818	1118118181	1811888811
128888888876	8188881181	8888111181	1188188118	1188818118	1888888188	1881111118	1881888111	8188118111
128888888876	1188888118	1888818118	8188888188	8888181181	1188111181	8881818111	1181188888	1881181111
128888888876	8888188188	1188181181	1188811181	1888888118	8188818118	1881881111	8188111118	8881888111
128888888876	1888811181	8188888118	8888118118	1188188188	1188881181	1181188111	1881118111	1881888888
128888888876	1188118118	1188888188	1888881181	8188811181	8888188118	8188188888	8881881111	1181111118
128888888876	8118181888	8811811888	8818888881	1118118888	1811811881	1111818811	1811188818	8111118818
128888888876	1118188881	1811818888	8118111881	8811881888	8818811888	8811118818	8888811811	1111881881
128888888876	8811811881	8818881888	1118111888	1811888881	8118118888	1811181881	8111118811	8811188818
128888888876	1811811888	8118188881	8811818888	8818811881	1118181888	8888888818	1111818818	1811111811
128888888876	8818818888	1118111881	1811881888	8118111888	8811888881	8111111811	8811181881	8888818811
128888888876	1881188811	1881881188	8188111188	8881881818	1181118181	8881181181	1118811181	1818188118
128888888876	8881811188	1181181818	1881118181	1881888811	8188181188	8118888118	8181818118	8881188188
128888888876	1881818181	8188188811	8881881188	1181111188	1881181818	1118888188	1818181181	8118811181
128888888876	1181181188	1881111188	1881881818	8188118181	8881888811	8181811181	8881188118	1118818118
128888888876	8188181818	8881818181	1181188811	1881181188	1881811188	1818118118	8118888188	8181881181
	128888888876	128888888876	128888888876	128888888876	128888888876	128888888876	128888888876	128888888876
	1	2	3	4	5	6	7	8



128888888876	128888888876	128888888876	128888888876	128888888876	128888888876	128888888876	128888888876	128888888876
1188818888	1888811881	1881118811	1881888818	8188118818	8881811811	1181181881	8881188811	1118881188
8888181888	1188111888	8881818818	1181111811	1881181881	1881818811	8188188818	8118811188	8181881818
1888888881	8188818888	1881881881	8188118811	8881888818	1181118818	1881111811	1118818181	1818188811
1188111881	1188881888	1181188818	1881118818	1881811811	8188181881	8881818811	8181881188	8881111188
8188811888	8888188881	8188111811	8881881881	1181118811	1881188818	1881818818	1818181818	8118818181
1118181818	1811818181	1111881181	1811111181	8111188118	8811118118	8888888188	8188811118	8888188111
8811888811	8818881188	8811188118	8888818118	1111888188	1811181181	8111111181	1188818111	1888888888
1811811188	8118181818	1811188188	8111181181	8811111181	8888888118	1111818118	8888181111	1188111118
8818818181	1118188811	8888811181	1111888118	1811118118	8111188188	8811181181	1888888111	8188818111
8118181188	8811811188	8111118118	8811188188	8888881181	1111811181	1811188118	1188188888	1188881111
8881888888	1181181111	8881181888	1118811888	1818188881	8118818888	8181811881	8118118811	8811888818
1881811118	8188188111	8118888881	8181818888	8881111881	1118881888	1818111888	1118118818	1811811811
1181118111	1881188888	1118811881	1818181888	8118811888	8181888881	8881118888	8811881881	8818818811
8188181111	8881811118	8181811888	8881188881	1118818888	1818111881	8118881888	1811888818	8118118818
1881188111	1881818111	1818118888	8118811881	8181881888	8881111888	1118888881	8818811811	1118181881
8811111811	8888881881	8188888811	8888181188	1188111188	1188881818	1888818181	1881181181	1881811181
1811118811	8111188818	1188811188	1888881818	8188818181	8888188811	1188181188	8881888118	1181118118
8888818818	1111811811	8888118181	1188188811	1188881188	1888811188	8188881818	1881888188	8188181181
8111181881	8811118811	1888881188	8188811188	8888181818	1188118181	1188888811	1181111181	1881188118
1111888818	1811118818	1188181818	1188818181	1888888811	8188881188	8888111188	8188118118	8881888188
8118818118	8181888188	8118111118	8811888111	8818818111	1118188888	1811881111	1111881888	1811111888
1118881181	1818111181	1118118111	1811888888	8118181111	8811811118	8818888111	8811188881	8888818888
8181888118	8881118118	8811881111	8818811118	1118188111	1811818111	8118188888	1811111881	8111181888
1818188188	8118881181	1811888111	8118118111	8811888888	8818881111	1118111118	8888811888	1111888881
8881111181	1118888118	8818888888	1118181111	1811811118	8118188111	8811818111	8111118888	8811111881
128888888876	128888888876	128888888876	128888888876	128888888876	128888888876	128888888876	128888888876	128888888876
9	10	11	12	13	14	15	16	17



12888888876	12888888876	12888888876	12888888876	12888888876	12888888876	12888888876	12888888876	12888888876
1818111188	8118881818	8181818181	8118181181	8811811181	8818888118	1118118118	1811888188	12888888876
8881118181	1118888811	1818181188	1118188118	1811818118	8118188188	8811881181	8818811181	12888888876
8118881188	8181811188	8881181818	8811888188	8818881181	1118111181	1811888118	8118118118	12888888876
1118881818	1818118181	8118888811	1811811181	8118188118	8811818118	8818888188	1118181181	12888888876
8181888811	8881181188	1118811188	8818818118	1118188188	1811881181	8118111181	8811888118	12888888876
1188118111	1188888888	1888881111	1881181888	1881811888	8188188881	8881818888	11811111881	12888888876
8188881111	8888111118	1188188111	8881888881	1181118888	18811111881	1881881888	81881111888	12888888876
1188888111	1888818111	8188888888	1881811881	8188181888	8881811888	1181188881	1881118888	12888888876
8888188888	1188181111	1188811118	1181111888	1881188881	1881818888	8188111881	8881881888	12888888876
1888811118	8188888111	8888118111	8188118888	8881811881	1181181888	1881111888	1881888881	12888888876
8818818818	1118111811	1811881881	1111888811	1811181188	8111111188	8811181818	8888818181	12888888876
8118181881	8811818811	8818888818	8811111188	8888881818	1111818181	1811188811	8111181188	12888888876
1118188818	1811818818	8118111811	1811118181	8111188811	8811181188	8888811188	1111881818	12888888876
8811811811	8818881881	1118118811	8888881188	1111811188	1811181818	8111118181	8811188811	12888888876
1811818811	8118188818	8811818818	8111181818	8811118181	8888888811	1111881188	1811111188	12888888876
8188188118	8881818118	1181188188	8881111118	1118888111	1818118111	8118888888	8181881111	12888888876
1881188188	1881881181	8188111181	8118818111	8181888888	8881181111	1118811118	1818188111	12888888876
8881811181	1181188118	1881118118	1118881111	1818111118	8118888111	8181818111	8881188888	12888888876
1881818118	8188188188	8881881181	8181888111	8881118111	1118888888	1818181111	8118811118	12888888876
1181181181	1881111181	1881888118	1818188888	8118881111	8181811118	8881188111	1118818111	12888888876
8111188881	8811118888	8888811881	8188818811	8888188818	1188118818	1188811811	1888881881	12888888876
1111811881	1811181888	8111111888	1188818818	1888811811	8188881881	8888118811	1188188818	12888888876
8811111888	8888888881	1111818888	8888181881	1188118811	1188888818	1888818818	8188811811	12888888876
1811118888	8111111881	8811181888	1888888818	8188818818	8888111811	1188181881	1188818811	12888888876
8888881888	1111811888	1811188881	1188111811	1188881881	1888818811	8188888818	8888118818	12888888876
12888888876	12888888876	12888888876	12888888876	12888888876	12888888876	12888888876	12888888876	12888888876
18	19	20	21	22	23	24	25	

It is based on 2 by 2 combinations of 25 numbers:

$\{11118, 11181, 11188, 11811, 11881, 11888, 18111, 18118, 18181, 18811, 18818, 18888, 81111, 81181, 81188, 81818, 81881, 81888, 88111, 88118, 88188, 88811, 88818, 88881, 88888\}$ .

These 25 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{11188, 88111\}$ ,  $\{18888, 88881\}$ ,  $\{11888, 88811\}$ ,  $\{81188, 88118\}$ ,  $\{81888, 88818\}$ ,  $\{11181, 18111\}$ ,  $\{11881, 18811\}$ ,  $\{18118, 81181\}$  and  $\{18818, 81881\}$ . The numbers  $\{11811, 18181, 81818, 88188, 88888\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.11 is **universal magic square**. Its



**magic sum** is

$$S_{25 \times 25} := 128888888876 = 100000 \times 1288876 + 1288876,$$

where

$$\begin{aligned} 1288876 := & 11118 + 11181 + 11188 + 11811 + 11881 + 11888 + 18111 + 18118 + 18181 + \\ & + 18811 + 18818 + 18888 + 81111 + 81181 + 81188 + 81818 + 81881 + \\ & + 81888 + 88111 + 88118 + 88188 + 88811 + 88818 + 88881 + 88888. \end{aligned}$$

It is **block-wise pandiagonal magic square** of order 25 where each block of order 5 is also a **pandiagonal** with different **magic sums**.

## 2.9.2 Bimagic Square

**Example 2.16.** *The magic square of order 25 for the digits 1 and 8 is given below in three parts:*

	pan	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195
10205205205195	11111111111	118811811818	181118118811	811818818181	818181181118	81118811118	818818118111	111181818818
10205205205195	811818118811	818181818181	11111181118	118811111111	181118811818	118111818818	181811181811	811118111181
10205205205195	118811181118	181118111111	811818811818	818181118811	111111818181	818818111181	11118181118	118111118111
10205205205195	818181811818	111111118811	118811818181	181118181118	811818111111	181811118111	811118818818	818818181811
10205205205195	181118818181	811818181118	818181111111	111111811818	118811118811	111181181811	118111111181	181811811118
10205205205195	818811118181	111118818118	118818181111	181181111818	811111811811	181818818811	811181181181	818111111118
10205205205195	181181181111	811111111818	818811811811	111118118181	118818818118	111811111118	118118811111	181818118818
10205205205195	111118811811	118818118181	181181818118	811111181111	818811111818	811181118818	818111818811	111811181181
10205205205195	811111818118	818811181111	111118111818	118818811811	181181118181	118118181181	181818111118	811181811111
10205205205195	118818111818	181181811811	811111118181	818811818118	111118181111	818111811111	111811118818	118118818811
10205205205195	8111118181818	818818111811	111181811181	118111118118	181811818111	118181111111	181111811818	811811118811
10205205205195	118111811181	181811118118	811118818111	818818181818	111181111811	818118118811	111818818181	118181181118
10205205205195	818818818111	111181181818	118111111811	181811811181	811118118118	181111181118	811811111111	818118811818
10205205205195	181811111811	811118811181	818818118118	111181818111	118111181818	111818811818	118181118811	181111818181
10205205205195	111181118118	118111818111	181811181818	811118111811	818818811181	811811818181	818118181118	111818111111
10205205205195	181818811118	811181118111	818111818818	111811181811	118118111181	111111118181	118811818118	181118181111
10205205205195	111811818818	118118181811	181818111181	811181811118	818111118111	811818181111	818181111818	111111811811
10205205205195	811181111181	818111811118	111811118111	118118818818	181818181811	118811811811	181118118181	811818818118
10205205205195	118118118111	181818818818	811181181811	818111111181	111811811118	818181818118	111111181111	118811111818
10205205205195	818111181811	111811111181	118118811118	181818118111	811181818818	181118111818	811818811811	818181118181
10205205205195	118181818811	181111181181	811811111118	818118811111	111818118818	818811181818	111118111811	118818811181
10205205205195	818118111118	111818811111	118181118818	181111818811	811811181181	181181811181	811111118118	818811818111
10205205205195	181111118818	811811818811	818118181181	111818111118	118181811111	111118818111	118818181818	181181111811
10205205205195	111818181181	118181111118	181111811111	811811118818	818118818811	811111111811	818811811181	111118118118
	811811811111	818118118818	111818818811	118181181181	181111111118	118818118118	181181818111	811111181818
	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195
	1	2	3	4	5	6	7	8



10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195
11811181811	18181111181	118181118181	18111818118	811811181111	818118111818	111818811811	818811818811	111118181181
818818811118	111181118111	818118181111	111818111818	118181811811	181111118181	811811818118	181181111118	811111811111
181811818818	811118181811	181111811811	811811118181	818118818118	111818181111	118181111818	111118118818	118818818811
111181111181	118111811118	111818818118	118181811111	181111111818	811811811811	818118118181	811111181181	818811111118
811118118111	818818818818	811811111818	818118811811	111818118181	118181818118	181111181111	118818811111	181181118818
111811811111	118118118818	111111181818	118811111811	181118811181	811818118118	818181818111	811118111111	818818811818
811181818811	818111181181	811818811181	818181118118	111111818111	118811181818	181118111811	118111118811	181811818181
118118111118	181818811111	118811818111	181118181818	811818111811	818181811181	111111118118	818818181118	111181111111
818111118818	111811818811	818181111811	111111811181	118811118118	181118818111	811818181818	181811811818	811118118811
181818181181	811181111118	181118118118	811818818111	818181181818	111111111811	118811811181	111181818181	118111181118
818118818181	111818181118	818811811118	111118118111	118818818818	181181181811	811111111181	181818118181	811181818118
181111111111	811811811818	181181818818	811111181811	818811111181	111118811118	118818118111	111811181111	118118111818
111818118811	118181818181	111118111181	118818811118	181181118111	811111818818	818811181811	811181811811	818111118181
811811181118	818118111111	811111118111	818811818818	111118181811	118818111181	181181811118	118118818118	181818181111
118181811818	181111118811	118818181811	181181111181	811111811118	818811118111	111118818818	818111111818	111811811811
811818111818	818181811811	811118818811	818818181181	111181111118	118111811111	181811118818	118181181818	181111111811
118811118181	181118818118	118111111118	181811811111	811118118818	818818818811	111181181181	818118811181	111818118118
818181181111	111111111818	818818118818	111181818811	118111181181	181811111118	811118811111	181111818111	811811181818
181118811811	811818118181	181811181181	811118111118	818818811111	111181118818	118111818811	111818111811	118181811181
111111818118	118811181111	111181811111	118111118818	181811818811	811118181181	818818111118	811811118118	818118818111
181181118118	811111818111	181818111111	811181811818	818111118811	111811818181	118118181118	111111811118	118811118111
111118181818	118818111811	111811118811	118118818181	181818181118	811181111111	818111811818	811818818818	818181181811
811111811181	818811118118	811181181118	818111111111	111811811818	118118118811	181818818181	118811111181	181118811118
118818818111	181181181818	118118811818	181818118811	811181818181	818111181118	111811111111	818181118111	111111818818
818811111811	111118811181	818111818181	111811181118	118118111111	181818811818	811181118811	181118181811	811818111181
10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195
9	10	11	12	13	14	15	16	17

10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195
118818111118	181181811111	811111118818	181818181818	811181111811	818111811181	111811118118	118118818111	10205205205195
818811118818	111118818811	118818181181	111811811181	118118118118	181818818111	811181181818	818111111811	10205205205195
181181181181	811111111118	818811811111	811181818111	818111181818	111811111811	118118811181	181818118118	10205205205195
111118811111	118818118818	181181818811	118118111811	181818811181	811181118118	818111818111	111811181818	10205205205195
811111818811	818811181181	111118111118	818111118118	111811818111	118118181818	181818111811	811181811181	10205205205195
111181118811	118111818181	181811181118	118181811118	181111118111	811811818818	818118181811	111818111181	10205205205195
811118181118	818818111111	111181811818	818118818818	111818181811	118181111181	181111811118	811811111811	10205205205195
118111811818	181811118811	811118818181	181111111181	811811811118	818118118111	111818818818	118181181811	10205205205195
818818818181	111181181118	118111111111	111818118111	118181818818	181111181811	811811111181	818118811118	10205205205195
181811111111	811118811818	818818118811	811811118181	818118111181	111818811118	118181118111	181111818818	10205205205195
818111181111	111811111818	118118811811	111111818811	118811181181	181118111118	811818811111	818181118818	10205205205195
181818811811	811181118181	818111818118	811818111118	818181811111	111111118818	118811818811	181118181181	10205205205195
111811818118	118118181111	181818111818	118811118818	181118818811	811818181181	818181111118	111111811111	10205205205195
811181111818	818111811811	111811118181	818181118181	111111111118	118811811111	181118118818	811818818811	10205205205195
118118118181	181818818118	811181181111	181118811111	811818118818	818181818811	111111181181	118811111118	10205205205195
811811811181	818118118118	111818818111	818811111111	111118811818	118818118811	181181818181	811111181118	10205205205195
118181818111	181111181818	811811111811	181181118811	811111818181	818811181118	111118111111	118818811818	10205205205195
818118111811	111818811181	118181118118	111118181118	118818111111	181181811818	811111118811	818811818181	10205205205195
181111118118	811811818111	818118181818	811111811818	818811118811	111118818181	118818181118	181181111111	10205205205195
111818181818	118181111811	181111811181	118818818181	181181181118	811111111111	818811811818	111118118811	10205205205195
181118818818	811818181811	818181111181	811118118181	818818818118	111181181111	118111111818	181811811811	10205205205195
111111111181	118811811118	181118118111	118111181111	181811111818	811118811811	818818118181	111181818118	10205205205195
811818118111	818181818818	111111181811	818818811811	111181118181	118111818118	181811181111	811118111818	10205205205195
118811181811	181118111181	811818811118	181811818118	811118181111	818818111818	111181811811	118111118181	10205205205195
818181811118	111111118111	118811818818	111181111818	118111811811	181811118181	811118818118	818818181111	10205205205195
10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195	10205205205195
18	19	20	21	22	23	24	25	

It is based on 2 by 2 combinations of 25 numbers of 6-digits:

$\{111111, 111118, 111181, 111811, 111818, 118111, 118118, 118181, 118811, 118818, 181111, 181118, 181181, 181811, 181818, 811111, 811118, 811181, 811811, 811818, 818111, 818118, 818181, 818811, 818818\}$ .

These 25 numbers are **upside-down** and **mirror looking** in pairs:  $\{111118, 811111\}$ ,  $\{111181, 181111\}$ ,  $\{111811, 118111\}$ ,  $\{111818, 818111\}$ ,  $\{118118, 811811\}$ ,  $\{811818, 818118\}$ ,  $\{818811, 118818\}$ ,  $\{181818, 818181\}$ ,  $\{118181, 181811\}$  and  $\{181118, 811181\}$ . The numbers  $\{111111, 818818, 181181, 811118, 118811\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.11 is



**universal magic square.** Its **magic sum** is

$$S_{25 \times 25} := 10205205205195 = 1000000 \times 10205195 + 10205195,$$

where

$$\begin{aligned} 10205195 := & 111111 + 111118 + 111181 + 111811 + 111818 + 118111 + 118118 + 118181 + 118811 + \\ & + 118818 + 181111 + 181118 + 181181 + 181811 + 181818 + 811111 + 811118 + \\ & + 811181 + 811811 + 811818 + 818111 + 818118 + 818181 + 818811 + 818818. \end{aligned}$$

Moreover the above 25 numbers make a **pandiagonal magic square** of order 5. See below

		2041039	2041039	2041039	2041039	2041039
	111111	118811	181118	811818	818181	2041039
2041039	811118	818818	111181	118111	181811	2041039
2041039	118181	181111	811811	818118	111818	2041039
2041039	818811	111118	118818	181181	811111	2041039
2041039	181818	811181	818111	111811	118118	2041039
	2041039	2041039	2041039	2041039	2041039	2041039

The magic square given in Example 2.16 is a **pandiagonal bimagic square** of order 25, where each block of order 5 is also a **pandiagonal** with equal **magic sums**. Below are **magic** and **bimagic square sums**:

$$S_{25 \times 25} := 10205205205195; \quad Sb_{25 \times 25} := 6937585302942336618971255; \quad \text{and} \quad S_{5 \times 5} := 2041041041039.$$

**Remark 2.3.** Whole the work is 10-digits distributions of two numbers 1 and 8. This is only work where we used 12-digits distributions of two numbers 1 and 8 in order to get a **pandiagonal bimagic square** of magic square 25 given in Example 2.16. The Example 2.15 is with 10-digits distributions of two numbers 1 and 8 is **pandiagonal bimagic square**. In both the examples the blocks of order 5 are pandigonal magic squares. In case of Example 2.15 these are of different magic sums while for the Example 2.16 these are of equal magic sums.

## 2.10 Magic Square of Order 26

**Example 2.17.** The *magic square* of order 26 for the digits 1 and 8 is given below in three parts:

26x26	mgc							
1888118881	1111818111	1188188818	1818881111	8118888881	8188111881	8118181811	8881188181	1188818811
1811111118	8111881118	8111118111	8818188818	8181118888	8118111181	1881888181	1811888118	1111818188
8881811881	1811181111	1118811188	1888818111	1818888818	8811888881	1811818881	1881118188	8188181188
8111118188	8881888181	1811118888	8811188111	8888118111	8181188818	8118811181	8188181118	1188181811
8888181188	1888881811	8881818188	1811188881	1188811888	1118118111	8811888818	8118118881	1881811188
1188181881	1118181181	8888188118	8881881811	1811111181	8881188811	1888118111	8118888818	1811881118
8181181181	8818118818	1888118118	1118181188	8881888118	1811118881	1111811118	8111818111	8118188818
8818188811	8811818118	1818818811	8111881881	1888181181	8881881188	1811181118	8111181111	1118818111
1881111888	1818811118	8118881881	8181111881	1118818818	8111818118	8881881181	1811111188	1888818888
1881888818	1188188811	8181181111	8118118818	8811888181	8811118811	1118881881	8881818118	1811188111
8881118111	1881188818	8818111118	8811818888	1811818811	8118818188	1188811881	8811118818	8881881881
8811181118	1188811188	8881188111	1111811888	8111188811	1888811118	8888181111	1118118888	1888188881
8118888118	8118181188	1811881181	8188118118	1881881881	1881118818	1188118811	8818111881	1818888181
8811811188	8118888111	8118111888	1811888811	8188111118	1881881111	1881118888	1188188881	8818111181
1188888111	8881111888	1111888811	8111111118	1888881111	8888118888	1118188881	1888111181	8111818881
8188181111	8888111818	1881881888	8118888181	1881111188	1888111888	8111118118	1111818811	8188888811
1888811818	8188181888	8811811881	1881881118	1118188111	1111881181	8881118818	8188811888	1181818118
1811881888	8181118811	8188118881	8888111188	8881181188	1188881881	8188888111	1181881181	1118188118
1818818818	1811811181	1888881118	1188888118	8811118118	8188811188	1181881188	8888181811	8118811888
8118188881	8111118881	8811181811	1118881181	8188881118	1181888118	1888818188	8811888111	8881111818
1111811181	1118818188	8111881188	8188818881	1181881811	8111188181	8181111188	1188811818	8811881888
8111888181	1888188118	8188811181	1181818188	1111811881	1818881118	8811111818	8181181888	1881181181
1118181811	8188888881	1181888181	8881118811	8818118881	1118811818	1818881888	1881881188	8181111118
8188818888	1181811881	1188818818	1188111181	8111811818	8818181888	8188188118	1818888811	8811181111
1181818811	8811181881	1881188881	1888111818	1188181888	1811881811	8818111888	1118811118	8888118818
1118818118	1881818888	1118111818	1881181888	8118118188	1188188111	8111888811	1888881881	8111111881
129999999987	129999999987	129999999987	129999999987	129999999987	129999999987	129999999987	129999999987	129999999987
1	2	3	4	5	6	7	8	9



8881818818	1811188811	8111888111	8811881188	1118888118	8811111888	8111181881	1181818888	8188818118
8881111881	8881818811	1118811888	8118881181	8811181188	1188888811	1181888881	8188881881	1881181811
8111181811	1111888181	8811188811	8118118118	1188881181	8881111118	8188818818	1188188118	1888181881
1881881181	1888888118	1188811118	1811881881	8881118118	1111881111	8818181188	8111818818	1118888881
8818188118	1881118118	8881181111	8188118818	1111881881	8111118888	1118818811	8811111181	8118888811
1881188111	1818881188	1111818888	1881818811	8111118818	1888888881	1188818881	8118111118	8188111888
8188111188	1188111888	8111188881	1881111881	1888818811	8888111181	1811881111	1881888811	8811181888
1811888818	1881888111	1888811181	1188188181	8888111881	1118118881	1881111118	1188881888	8118111818
8811118111	8188188818	8888118881	8818118188	1118188181	1888181118	8881181888	1811811818	8811811181
8888188881	1188818111	1118181118	1818881811	1888118188	8111811188	8188111818	8118818881	8881181181
1811111888	1118111181	1888111188	8181188118	8111881811	1118888111	8118181118	1111818118	1181881111
8111811181	1118818881	1811118111	1181881888	8881811818	8188888818	8811881181	8181181188	1818888118
8181118188	8811881811	8188811818	8881888818	1181818111	1811181888	1888188111	1118111188	8888181118
1818818881	8181181118	1181888818	1811111818	8188881888	8881818111	8111888118	1888181811	1118118188
1118881118	8811111188	8881881888	8188818111	1811188818	1181811818	8118811881	8811818811	8181118818
1181881881	8111881181	8118188118	8811118881	8811881118	1188181188	8181181811	1811111881	8881811118
1888181188	1811811118	8118881811	1118811181	8181118881	1881188118	1188118111	1818818188	1811118811
8118188811	8111111818	8811818188	8111888881	1818811181	1881881811	1111888818	1881118111	8818188181
1111811818	8118181888	8181188181	1888118888	8818188881	8188118188	1118188811	8881188818	1881818111
8118881888	8818181881	1818811881	1118181111	1188118888	1811888181	1881811181	8888111888	1188888818
1188118118	8118818888	8818118811	8888111118	1881181111	8118111881	1818811888	8188188881	1888888111
8811881111	8881188881	1188118818	1888888811	1881811118	8118818811	8888118118	8818188111	1811818888
1188818888	1888111881	1881181881	8111111888	8188188811	8811818818	8811118188	1888881181	1188111188
1118118811	8888118188	1881818118	1111888111	1811811888	8181181881	1888811188	1118888181	8111181188
1888888181	8188881111	8188181181	8881111188	8118188111	1818818118	8881818888	8111181118	8111811881
8188811118	1181818818	1811881188	1188881118	8118811188	8818181181	1811188181	8881881111	1111818881
129999999987	129999999987	129999999987	129999999987	129999999987	129999999987	129999999987	129999999987	129999999987
10	11	12	13	14	15	16	17	18

								129999999987
1881818188	8888181181	1118111118	8818181118	8181111181	1888881888	1881111818	1811811188	129999999987
1118118118	1888181111	1818811188	8811818881	8888181888	1188111818	8188188111	1888818818	129999999987
8111818888	8181188111	8118881118	1118181888	8818111818	1881811888	8888118811	1181811181	129999999987
8811811888	8118111188	1888181888	1818811818	1881188811	1118111881	1181818881	8188818811	129999999987
1811888111	8111881888	8181111818	1188111118	1888188181	1181881118	8188811881	1818881181	129999999987
1118881888	8811811818	8818181111	8111818188	1181811188	8188888181	8181118118	8811111881	129999999987
8118811818	1818818888	1118881811	1181888111	8188818188	8811881881	1188888181	8881181118	129999999987
8181188881	8811188118	1181811888	8188881811	8118818818	8881118188	1111811188	8188118888	129999999987
1188881188	1181888811	8188888118	8118118811	1111881811	8111188111	1881888881	1188181111	129999999987
1181811118	8188881188	1811811881	8111188118	1888811888	1881111181	8818118888	1111881888	129999999987
8188881181	8188188181	1888881188	8888188811	1188118881	1818888881	8111181888	1881811818	129999999987
8818181811	1188118188	1881188181	1881811881	8188118811	1811818818	8118181881	8118818118	129999999987
1888818881	8111111181	1111888881	8881118888	1188881111	8811111118	1118888811	8111811888	129999999987
8888188181	1888811881	8111118811	1111818818	8881181881	1188818118	8811181181	1118881188	129999999987
1818881881	8818118118	1188181181	1881181188	1881888118	8188181811	1811818188	8118188181	129999999987
8881111181	1118118818	1188818188	1811888881	1818888111	1118818888	1888888818	8818118111	129999999987
8881888811	1188888881	8888181881	8811188181	8118118888	8818111188	8111881111	8111188818	129999999987
1811118818	8881811888	8811118888	1888818118	1118811881	8118881111	1188181118	1888111118	129999999987
1188111881	1811181881	8881888111	1118881111	8111181181	8111818811	8811811118	1881118881	129999999987
8188118111	1881118811	1811118118	8881811188	8111811118	1111881188	1888118818	8181188811	129999999987
8811188818	1811818111	1881818818	1811181181	8881881118	1888188811	8881188118	1118181881	129999999987
8111111188	1118888818	8118118111	8188181881	1811181188	8881818881	1118111888	1188881811	129999999987
8118181111	1111881118	8111888818	8118818111	1811818118	1811188118	8881811181	8888188111	129999999987
1881181118	8118811118	8881118881	1888188818	8811818111	8118181181	1811181811	8881888881	129999999987
1111888118	1881818881	8811888811	1188811181	1118188818	8181118111	8118881188	1811118188	129999999987
1888118811	8881181811	8188111181	8181111888	8811188881	8888188818	1818818111	8811888118	129999999987
129999999987	129999999987	129999999987	129999999987	129999999987	129999999987	129999999987	129999999987	129999999987
19	20	21	22	23	24	25	26	

It is based on 2 by 2 combinations of 26 numbers:

$\{11118, 11181, 11188, 11818, 11881, 11888, 18111, 18118, 18188, 18811, 18818, 18881, 18888, 81111, 81118, 81181, 81188, 81811, 81881, 81888, 88111, 88118, 88181, 88811, 88818, 88881\}$ .

These 26 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{11181, 18111\}$ ,  $\{11188, 88111\}$ ,  $\{11818, 81811\}$ ,  $\{11881, 18811\}$ ,  $\{11888, 88811\}$  ,  $\{18118, 81181\}$ ,  $\{18188, 88181\}$  ,  $\{18818, 81881\}$ ,  $\{18888, 88881\}$ ,  $\{81188, 88118\}$  and  $\{81888, 88818\}$ . The numbers  $\{18881, 81118\}$  are already **upside-down**



and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.17 is **universal magic square**. Its **magic sum** is

$$S_{26 \times 26} := 129999999987 = 100000 \times 1299987 + 1299987,$$

where

$$\begin{aligned} 1299987 := & 11118 + 11181 + 11188 + 11818 + 11881 + 11888 + 18111 + 18118 + 18188 + \\ & + 18811 + 18818 + 18881 + 18888 + 81111 + 81118 + 81181 + 81188 + 81811 + \\ & + 81881 + 81888 + 88111 + 88118 + 88181 + 88811 + 88818 + 88881. \end{aligned}$$

## 2.11 Magic Square of Order 27

### 2.11.1 Without Block

**Example 2.18.** *The magic square of order 27 for the digits 1 and 8 is given below in three parts:*

27x27	mgc							
1888118881	8111888881	1118811881	8811188818	1188881881	8881188111	1111881188	8111111118	1888881811
8888181118	1118111181	1811118811	8881881888	8188818118	1181811888	8181188118	1818881111	8818118188
1188111188	1881118111	1881818818	8188111818	1811881181	8118188811	8118888888	8811818888	8888888181
8881888111	8188888818	1181881881	8181181811	1818881188	8818111118	8888111881	1118118881	1811188881
8188111888	1811881888	8118118118	8118818188	8811888118	8888881111	1188118811	1881181118	1881811181
8811188811	1188811818	8881181181	1111888181	8111188888	1888818888	1888118818	8111811188	1118818111
8118811118	8811881811	8888881188	1188188881	1881111881	1881818881	8188181881	1811888111	8118188818
1111881111	8111118188	1888888118	1888111181	8111818811	1118881118	8811118118	1188811888	8881181888
8181118888	1818888181	8818188888	8888118111	1118118818	1811111188	8881881181	8188888811	1181811818
1188888881	8881111881	8811118881	8111181881	1888888111	1111888818	8111811118	1118881811	1888181188
8188811181	1181818811	8881881118	1818818118	8818111888	8181181888	1118181111	1811118188	8888188118
1811818111	8118118818	8188111188	8811881181	8888888811	8118811818	1881118888	1881888181	1188188888
1818888818	8818181881	8181188111	1118181188	1811111118	8888181811	8188818881	1181888881	8881811881
8811881888	8888818118	8118811888	1881188118	1881881111	1188118188	1811881118	8118111181	8188118811
8111111818	1888881181	1111888811	8111888888	1118818888	1888188181	1188811188	8881118111	8811118818
1881181811	1881881188	1188111118	1811811881	8118118881	8188188881	8811888111	8888888818	8118881881
8111818188	1118888118	1888181111	1188818811	8881181118	8811111181	8111111888	1888881888	1111818118
1118188181	1811188888	8888118888	8188818818	1181811188	8881818111	1818888811	8818111818	8181181181
1888811881	1111818881	8111188881	1118888111	1888188818	8111881881	8881181811	8811181188	1188811118
8818118811	8181181118	1818811181	1811111888	8888181888	1118118118	1181818188	8881888118	8188881111
8888818818	8118811188	8811818111	1881888811	1188111818	1881181181	8118188181	8188188888	1811818888
1811181881	8888188111	1118188818	1181811118	8881881811	8188881188	8818188881	8181111881	1818818881
1881818118	1188111888	1881181888	8118181111	8188118188	1811888118	8888811181	8118818811	8811881118
1118881181	1888188811	8111811818	8881118888	8811188181	1188888888	1888818111	1111818818	8111111188
8118181188	8188111118	1811881811	8888818881	8118888881	8811811881	1881888818	1188181881	1881188111
8881188118	8811181111	1188818188	1888881118	1111811181	8111118811	1118881888	1888118118	8111811888
1181888888	8881818888	8188888181	8818111188	8181118111	1818818818	1811111818	8888181181	1118188811
138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875
1	2	3	4	5	6	7	8	9



8888111888	1118181888	1811118118	8881818188	8188888118	1181881111	8181118811	1818881118	8818111181
1188188811	1881111818	1881881181	8188188181	1811888888	8118118888	8118818818	8811811188	8888818111
1888188111	8111888818	1118881881	8811181811	1188881188	8881111118	1111811881	8111118881	1888888881
8188181111	1811818188	8118188118	8118811181	8811818811	8888881118	1188118118	1881111888	1881881888
8811118888	1188888181	8881188888	1111818111	8111118818	1888811188	1888181181	8111888811	1118811818
8881811118	8188881811	1181881188	8181188881	1818811881	8818118881	8888181881	1118188111	1811188818
1111881118	8111111181	1888818811	1888181888	8111818118	1118811888	8811188118	1188881111	8881118188
8181111188	1818818111	8818118818	8888111818	1118181181	1811188811	8881888888	8188818888	1181888181
8118818881	8811888881	8888811881	1188188818	1881181881	1881888111	8188181188	1811811118	8118181811
8188881888	1181818118	8881811888	1818888118	8818181111	8181118188	1118181118	1811111181	8888118811
1811811818	8118181181	8188188811	8811888888	8888818888	8118888181	1881111188	1881818111	1188118818
1188888818	8881181881	8811188111	8111181188	1888811118	1111881811	8111818881	1118888881	1888111881
8811818188	8888888118	8118881111	1881118811	1881881118	1188111181	1811811888	8118181888	8188118118
8111188181	1888888888	1111818888	8111818818	1118811188	1888118111	1188888811	8881111818	8811181181
1818881811	8818181188	8181111118	1118111881	1811118881	8888188881	8188888111	1181888818	8881881881
8111811181	1118818811	1888181118	1188818118	8881111888	8811181888	8111181111	1888818188	1111888118
1118118111	1811118818	8888111188	8188881181	1181888811	8881811818	1818818888	8818188181	8181188888
1881188881	1881811881	1188118881	1811881881	8118188111	8188188818	8811811118	8888881811	8118881188
8818118118	8181111888	1818881888	1811181111	8888118188	1118188118	1181811181	8881818811	8188881118
8888881181	8118888811	8811811818	1881818888	1188188181	1881188888	8118118111	8188118818	1811811188
1888881881	1111888111	8111188818	1118811118	1888181811	8111881188	8881188881	8811111881	1188818881
1881888118	1188181111	1881118188	8118181118	8188111181	1811818811	8888881888	8118818118	8811811888
1118888888	1888118888	8111888181	8881111188	8811118111	1188818818	1888811818	1111881181	8111188811
1811181188	8888111118	1118181811	1181818881	8881888881	8188811881	8818188818	8181181881	1818888111
8881118811	8811181118	1188811181	1888811888	1111881888	8111118118	1118818188	1888188118	8111881111
1181818818	8881811188	8188818111	8818188811	8181111818	1818881181	1811188181	8888188888	1118118888
8118111881	8188118881	1811888881	8888888111	8118888818	8811881881	1881881811	1188181188	1881111118
138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875
10	11	12	13	14	15	16	17	18



									13888888875
1188118888	1881188181	1881888888	8188118111	1811818818	8118111188	8118881181	8811888811	8888811818	13888888875
1888111118	8111881811	1118881188	8811188881	1188811881	8881118881	1111881881	8111188111	1888888818	13888888875
8888181111	1118118188	1811188118	8881811181	8188818811	1181881118	8181118118	1818811888	8818181888	13888888875
8811111188	1188818111	8881118818	1111811818	8111181181	1888888811	1888188888	8111818888	1118888181	13888888875
8881818881	8188888881	1181811881	8181188818	1818881881	8818188111	8888181188	1118111118	1811181811	13888888875
8188181118	1811811181	8118118811	8118881888	8811818118	8888811888	1188188118	1881181111	1881818188	13888888875
8181188811	1818811818	8818181181	8888188181	1118188888	1811118888	8881818818	8188811188	1181818111	13888888875
8118888111	8811888818	8888881881	1188181811	1881181188	1881811118	8188111881	1811818881	8118188881	13888888875
1111811888	8111181888	1888818118	1888118188	8111888118	1118881111	8811118811	1188881118	8881111181	13888888875
1811888181	8118188888	8188118888	8811818818	8888811188	8118818111	1881188811	1881811818	1188181181	13888888875
1188881811	8881181188	8811111118	8111111881	1888818881	1111888881	8111888111	1118888818	1888181881	13888888875
8188818188	1181888118	8881881111	1818818811	8818181118	8181111181	1118111888	1811181888	8888118118	13888888875
8111118111	1888818818	1111811188	8111881181	1118888811	1888111818	1188818888	8881188181	8811188888	13888888875
1818888881	8818111881	8181118881	1118181881	1811188111	8888188818	8188811118	1181881811	8881881188	13888888875
8811811181	8888818811	8118881118	1881118118	1881811888	1188181888	1811881111	8118118188	8188188118	13888888875
1118111818	1811181181	8888188811	8188888888	1181818888	8881888181	1818811188	8818118111	8181118818	13888888875
1881188818	1881881881	1188188111	1811881188	8118111118	8188181811	8811818881	8888888881	8118811881	13888888875
8111881888	1118818118	1888111888	1188888118	8881181111	8811118188	8111181118	1888811181	1111818811	13888888875
8888888888	8118818888	8811888181	1881811188	1188118111	1881118818	8118111818	8188181181	1811888811	13888888875
1888881188	1111811118	8111181811	1118818881	1888188881	8111811881	8881188818	8811181881	1188888111	13888888875
8818188118	8181181111	1818818188	1811181118	8888111181	1118118811	1181881888	8881818118	8188811888	13888888875
1118818818	1888111188	8111818111	8881188811	8811111818	1188881181	1888888181	1111888888	8111118888	13888888875
1811111881	8888118881	1118188881	1181888111	8881888818	8188881881	8818181811	8181181188	1818811118	13888888875
1881818811	1188181118	1881111181	8118111888	8188181888	1811818118	8888818188	8118888118	8811881111	13888888875
1181881181	8881888811	8188811818	8818118888	8181188181	1818888888	1811118111	8888118818	1118111188	13888888875
8118181881	8188188111	1811888818	8888811118	8118881811	8811881188	1881888881	1188111881	1881118881	13888888875
8881118118	8811111888	1188881888	1888881111	1111818188	8111188118	1118811181	1888118811	8111881118	13888888875
13888888875	13888888875	13888888875	13888888875	13888888875	13888888875	13888888875	13888888875	13888888875	13888888875
19	20	21	22	23	24	25	26	27	

It is based on 2 by 2 combinations of 27 numbers:

$\{11118, 11181, 11188, 11818, 11881, 11888, 18111, 18118, 18188, 18811, 18818, 18881, 18888, 81111, 81118, 81181, 81188, 81811, 81881, 81888, 88111, 88118, 88181, 88811, 88818, 88881, 88888\}$ .

These 27 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}, \{11181, 18111\}, \{11188, 88111\}, \{11818, 81811\}, \{11881, 18811\}, \{11888, 88811\}, \{18118, 81181\}, \{18188, 88181\}, \{18818, 81881\}, \{18888, 88881\},$



$\{81188, 88118\}$  and  $\{81888, 88818\}$ . The numbers  $\{18881, 81118, 88888\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.18 is **universal magic square**. Its **magic sum** is

$$S_{27 \times 27} := 138888888875 = 100000 \times 1388875 + 1388875,$$

where

$$\begin{aligned} 1388875 := & 11118 + 11181 + 11188 + 11818 + 11881 + 11888 + 18111 + 18118 + 18188 + \\ & + 18811 + 18818 + 18881 + 18888 + 81111 + 81118 + 81181 + 81188 + 81811 + \\ & + 81881 + 81888 + 88111 + 88118 + 88181 + 88811 + 88818 + 88881 + 88888. \end{aligned}$$

## 2.11.2 Block of Order 9

**Example 2.19.** *The semi-magic square of order 27 for the digits 1 and 8 is given below in three parts:*

27x27	semi-mgc							
1118188811	8118118881	8188811888	8118888111	1881181111	8111811188	1881888881	8881818888	1811111118
8111811118	8118888881	1881118888	1811111888	1881888811	8881818881	8188811188	1118188111	8118181111
8881881111	1811111188	1881888111	8118118888	8188811118	1118188881	1881118881	8111811888	8118888811
1811118888	1881811118	8881888881	8188818881	1118111888	8118188811	8111881111	8118811188	1881188111
8118188111	8188881111	1118111188	1881188881	8111818888	8118811118	8881888811	1811118881	1881811888
8118811888	1881188811	8111818881	1881811188	8881888111	1811181111	1118111118	8118188881	8188818888
1881111188	8111888111	8118881111	8881811118	1811188881	1881818888	8118111888	8188888811	1118118881
1881818881	8881811888	1811188811	1118181111	8118111188	8188888111	8118818888	1881111118	8111888881
8188888881	1118118888	8118111118	8111888811	8118818881	1881111888	1811188111	1881881111	8881811188
1111888181	8111118188	8881118118	8811188118	1188881881	1888811881	1888188888	8888181811	1118811818
1888811818	8811188888	1188881811	1118818118	1888188181	8888118188	8881111881	1111888118	8111181881
8888181881	1118811881	1888188118	8111181811	8881111818	1111888888	1188818188	1888818118	8811188181
1118881811	1888111818	8888188888	8881118188	1111818118	8111188181	1888881881	8811111881	1188888118
8111188118	8881181881	1111811881	1188888888	1888881811	8811111818	8888188181	1118818188	1888118118
8811118118	1188888181	1888818188	1888111881	8888188118	1118881881	1111811818	8111188888	8881181811
1188811881	1888888118	8811181881	8888111818	1118888888	1888181811	8111118118	8881188181	1111818188
1888118188	8888118118	1118888181	1111881881	8111111881	8881188118	8811181811	1188811818	1888888888
8881188888	1111881811	8111111818	1888888181	8811118188	1188818118	1118888118	1888181881	8888111881
1181881888	8188118818	8818118811	8811881188	1811881181	8181118111	1818888818	8888881118	1188111181
8181111181	8811888818	1811881118	1188118811	1818881888	8888818818	8818118111	1181881188	8188181181
8888881181	1188118111	1818881188	8188181118	8818111181	1181888818	1811818818	8181118811	8811881888
1188181118	1818811181	8888888818	8818118818	1181818811	8188181888	8181181181	8811818111	1811881188
8188181188	8818181181	1181818111	1811888818	8181181118	8811811181	8888881888	1188118818	1818818811
8811818811	1811881888	8181118818	1818818111	8888881188	1188181181	1181811181	8188188818	8818181118
1811818111	8181181188	8811881181	8888811181	1188188818	1818881118	8188118811	8818181888	1181818818
1818818818	8888818811	1188181888	1181881181	8188118111	8818181188	8811881118	1811811181	8181188818
8818188818	1181881118	8188111181	8181181888	8811818818	1811818811	1188181188	1818881181	8888818111
138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875
1	2	3	4	5	6	7	8	9



1181888181	8188118188	8818118118	8811888118	1811881881	8181111881	1818888888	8888881811	1188111818
8181111818	8811888888	1811881811	1188118118	1818888181	8888818188	8818111881	1181888118	8188181881
8888881881	1188111881	1818888118	8188181811	8818111818	1181888888	1811818188	8181118118	8811888181
1188181811	1818811818	8888888888	8818118188	1181818118	8188188181	8181181881	8811811881	1811888118
8188188118	8818181881	1181811881	1811888888	8181181811	8811811818	8888888181	1188118188	1818818118
8811818118	1811888181	8181118188	1818811881	8888888118	1188181881	1181811818	8188188888	8818181811
1811811881	8181188118	8811881881	8888811818	1188188888	1818881811	8188118118	8818188181	1181818188
1818818188	8888818118	1188188181	1181881881	8188111881	8818188118	8811881811	1811811818	8181188888
8818188888	1181881811	8188111818	8181188181	8811818188	1811818118	1188188118	1818881881	8888811881
1118181888	8118118818	8188818811	8118881188	1881181181	8111818111	1881888818	8881881118	1811111181
8111811181	8118888818	1881181118	1811118811	1881881888	8881818818	8188818111	1118181188	8118181181
8881881181	1811118111	1881881188	8118181118	8188811181	1118188818	1881118818	8111818811	8118881888
1811181118	1881811181	8881888818	8188818818	1118118811	8118181888	8111881181	8118818111	1881181188
8118181188	8188881181	1118118111	1881188818	8111881118	8118811181	8881881888	1811118818	1881818811
8118818811	1881181888	8111818818	1881818111	8881881188	1811181181	1118111181	8118188818	8188881118
1881118111	8111881188	8118881181	8881811181	1811188818	1881881118	8118118811	8188881888	1118118818
1881818818	8881818811	1811181888	1118181181	8118118111	8188881188	8118881118	1881111181	8111888818
8188888818	1118181118	8118111181	8111881888	8118818818	1881118811	1811181188	1881881181	8881818111
1111888811	8111118881	8881111888	8811188111	1188881111	1888811188	1888188881	8888118888	1118811118
1888811118	8811188881	1188818888	1118811888	1888188811	8888118881	8881111188	1111888111	8111181111
8888181111	1118811188	1888188111	8111118888	8881111118	1111888881	1188818881	1888811888	8811188811
1118818888	1888111118	8888188881	8881118881	1111811888	8111188811	1888881111	8811111188	1188888111
8111188111	8881181111	1111811188	1188888881	1888818888	8811111118	8888188811	1118818881	1888111888
8811111888	1188888811	1888818881	1888111188	8888188111	1118881111	1111811118	8111188881	8881118888
1188811188	1888888111	8811181111	8888111118	1118888881	1888118888	8111111888	8881188811	1111818881
1888118881	8888111888	1118888811	1111881111	8111111188	8881188111	8811118888	1188811118	1888888881
8881188881	1111818888	8111111118	1888888811	8811118881	1188811888	1118888111	1888181111	8888111188
138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875
10	11	12	13	14	15	16	17	18

									138888943342
1111881888	8111118818	8881118811	8811181188	1188881181	1888818111	1888188818	8888181118	1118811181	138888888875
1888811181	8811188818	1188881118	1118818811	1888181888	8888118818	8881118111	1111881188	8111181181	138888888875
8888181181	1118818111	1888181188	8111181118	8881111181	1111888818	1188818818	1888818811	8811181888	138888888875
1118881118	1888111181	8888188818	8881118818	1111818811	8111181888	1888881181	8811118111	1188881188	138888888875
8111181188	8881181181	1111818111	1188888818	1888881118	8811111181	8888181888	1118818818	1888118811	138888888875
8811118811	1188881888	1888818818	1888118111	8888181188	1118881181	1111811181	8111188818	8881181118	138888888875
1188818111	1888881188	8811181181	8888111181	1118888818	1888181118	8111118811	8881181888	1111818818	138888888875
1888118818	8888118811	1118881888	1111881181	8111118111	8881181188	8811181118	1188811181	1888888818	138888888875
8881188818	1111881118	8111111181	1888881888	8811118818	1188818811	1118881188	1888181181	8888118111	138888888875
1181888811	8188118881	8818111888	8811888111	1811881111	8181111188	1818888881	8888818888	1188111118	138888888875
8181111118	8811888881	1811818888	1188111888	1818888811	8888818881	8818111188	1181888111	8188181111	138888888875
8888881111	1188111188	1818888111	8188118888	8818111118	1181888881	1811818881	8181111888	8811888811	138888888875
1188118888	1818811118	8888888881	8818118881	1181811888	8188188811	8181181111	8811811188	1811888111	138888888875
8188188111	8818181111	1181811188	1811888881	8181118888	8811811118	8888888811	1188118881	1818811888	138888888875
8811811888	1811888811	8181118881	1818811188	8888888111	1188181111	1181811118	8188188881	8818118888	138888888875
1811811188	8181188111	8811881111	8888811118	1188188881	1818818888	8188111888	8818188811	1181818881	138888888875
1818818881	8888811888	1188188811	1181881111	8188111188	8818188111	8811818888	1811811118	8181188881	138888888875
8818188881	1181818888	8188111118	8181188811	8811818881	1811811888	1188188111	1818881111	8888811188	138888888875
1118188181	8118118188	8188818118	8118888118	1881181881	8111811881	1881888888	8881881811	1811111818	138888888875
8111811818	8118888888	1881181811	1811118118	1881888181	8881818188	8188811881	1118188118	8118181881	138888888875
8881881881	1811111881	1881888118	8118181811	8188811818	1118188888	1881118188	8111818118	8118888181	138888888875
1811181811	1881811818	8881888888	8188818188	1118118118	8118188181	8111881881	8118811881	1881188118	138888888875
8118188118	8188881881	1118111881	1881188888	8111881811	8118811818	8881888181	1811118188	1881818118	138888888875
8118818118	1881188181	8111818188	1881811881	8881888118	1811181881	1118111818	8118188888	8188881811	138888888875
1881111881	8111888118	8118881881	8881811818	1811188888	1881881811	8118118118	8188888181	1118118188	138888888875
1881818188	8881818118	1811188181	1118181881	8118111881	8188888118	8118881811	1881111818	8111888888	138888888875
8188888888	1118181811	8118111818	8111888181	8118818188	1881118118	1811188118	1881881881	8881811881	138888888875
138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	138888888875	144335588875
19	20	21	22	23	24	25	26	27	

This magic square is constructed with the same entries as of Example 2.18. These numbers lead us to a **universal semi-magic square** with **semi-magic sum** i.e.,

$$S_{27 \times 27} := 138888888875 = 100000 \times 1388875 + 1388875,$$

By the semi-magic square, it is understood that the sum of rows and columns is the same, while the principal diagonals sum is different. The magic square given in Example 2.19 is constructed based on blocks of order



9. These 9 blocks of order 9 are **magic squares** with different **magic sums** resulting in a **semi-magic square** of order 27.

## 2.12 Magic Square of Order 28

### 2.12.1 Blocks of Order 4

**Example 2.20.** *The magic square of order 28 for the digits 1 and 8 is given below in three parts:*

28x28	pan	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986
	1188888111	8811188811	1118811188	8881111888	1188881111	8811188881	1118811118	8881118888	1188881888
13999999986	1118811888	8881111188	1188888811	8811188111	1118818888	8881111118	1188888881	8811181111	1118818111
13999999986	8881188811	1118888111	8811111888	1188811188	8881188881	1118881111	8811118888	1188811118	8881188818
13999999986	8811111188	1188811888	8881188111	1118888811	8811111118	1188818888	8881181111	1118888881	8811111181
13999999986	1888888111	8111188811	1111811188	8888111888	1888881111	8111188881	1111811118	8888118888	1888881888
13999999986	1111811888	8888111188	1888888811	8111188111	1111818888	8888111118	1888888881	8111181111	1111818111
13999999986	8888188811	1111888111	8111111888	1888811188	8888188881	1111881111	8111118888	1888811118	8888188818
13999999986	8111111188	1888811888	8888188111	1111888811	8111111118	1888818888	8888181111	1111888881	8111111181
13999999986	1811188111	8188888811	1118111188	8881811888	1811181111	8188888881	1118111118	8881818888	1811181888
13999999986	1118111888	8881811188	1811188811	8188888111	1118118888	8881811118	1811188881	8188881111	1118118111
13999999986	8881888811	1118188111	8188811888	1811111188	8881888881	1118181111	8188818888	1811111118	8881888818
13999999986	8188811188	1811111888	8881888111	1118188811	8188811118	1811118888	8881881111	1118188881	8188811181
13999999986	1881188111	8118888811	1188111188	8811811888	1881181111	8118888881	1188111118	8811818888	1881181888
13999999986	1188111888	8811811188	1881188811	8118888111	1188118888	8811811118	1881188881	8118881111	1188118111
13999999986	8811888811	1188188111	8118811888	1881111188	8811888881	1188181111	8118818888	1881111118	8811888818
13999999986	8118811188	1881111888	8811888111	1188188811	8118811118	1881118888	8811881111	1188188881	8118811181
13999999986	1818888111	8181188811	1181811188	8818111888	1818881111	8181188881	1181811118	8818118888	1818881888
13999999986	1181811888	8818111188	1818888811	8181188111	1181818888	8818111118	1818888881	8181811111	1181818111
13999999986	8818188811	1181888111	8181111888	1818811188	8818188881	1181881111	8181118888	1818811118	8818188818
13999999986	8181111188	1818811888	8818188111	1181888811	8181111118	1818818888	8818181111	1181888881	8181111181
13999999986	1881888111	8118888811	1818111188	8181811888	1888181111	8111888881	1818111118	8181818888	1888181888
13999999986	1818111888	8181811188	1888188811	8111888111	1818118888	8181811118	1888188881	8111881111	1818118111
13999999986	8181888811	1818188111	8111811888	1888111188	8181888881	1818181111	8111818888	1888111118	8181888818
13999999986	8111811188	1888111888	8181888111	1818188811	8111811118	1888118888	8181881111	1818188881	8111811181
	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986
	1	2	3	4	5	6	7	8	9



13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986
8811188818	1118811181	8881118111	1188881188	8811188118	1118811881	8881118811	1188881811	8811188181	1118811818	8881118188
8881111181	1188888818	8811181888	1118818811	8881111881	1188888118	8811181188	1118818188	8881111818	1188888181	8811181811
1118881888	8811118111	1188811181	8881188118	1118881188	8811118811	1188811881	8881188181	1118881811	8811118188	1188811818
1188818111	8881181888	1118888818	8811111881	1188818811	8881181188	1118888118	8811111818	1188818188	8881181811	1118888181
8111188818	1111811181	8888118111	1888881188	8111188118	1111811881	8888118811	1888881811	8111188181	1111811818	8888118188
8888111181	1888888818	8111181888	1111818811	8888111881	1888888118	8111181188	1111818188	8888111818	1888888181	8111181811
1111881888	8111118111	1888811181	8888188118	1111881188	8111118811	1888811881	8888188181	1111881811	8111118188	1888811818
1888818111	8888181888	1111888818	8111111881	1888818811	8888181188	1111888118	8111111818	1888818188	8888181811	1111888181
8188888818	1118111181	8881818111	1811181188	8188888118	1118111881	8881818811	1811181811	8188888181	1118111818	8881818188
8881811181	1811188818	8188881888	1118118811	8881811881	1811188118	8188881188	1118118188	8881811818	1811188181	8188881811
1118181888	8188818111	1811111181	8881888118	1118181188	8188818811	1811111881	8881888181	1118181811	8188818188	1811111818
1811118111	8881881888	1118188818	8188811881	1811118811	8881881188	1118188118	8188811818	1811118188	8881881811	1118188181
8118888818	1188111181	8811818111	1881181188	8118888118	1188111881	8811818811	1881181811	8118888181	1188111818	8811818188
8811811181	1881188818	8118881888	1188118811	8811811881	1881188118	8118881188	1188118188	8811811818	1881188181	8118881811
1188181888	8118818111	1881111181	8811888118	1188181188	8118818811	1881111881	8811888181	1188181811	8118818188	1881111818
1881118111	8811881888	1188188818	8118811881	1881118811	8811881188	1188188118	8118811818	1881118188	8811881811	1188188181
8181188818	1181811181	8818118111	1818881188	8181188118	1181811881	8818118811	1818881811	8181188181	1181811818	8818118188
8818111181	1818888818	8181181888	1181818811	8818111881	1818888118	8181181188	1181818188	8818111818	1818888181	8181181811
1181881888	8181118111	1818811181	8818188118	1181881188	8181118811	1818811881	8818188181	1181881811	8181118188	1818811818
1818818111	8818181888	1181888818	8181111881	1818818811	8818181188	1181888118	8181111818	1818818188	8818181811	1181888181
8118188818	1811811181	8181818111	1888181188	811888118	1818111881	8181818811	1888181811	811888181	1818111818	8181818188
8181811181	1888188818	8111881888	1818118811	8181811881	1888188118	8111881188	1818118188	8181811818	1888188181	8111881811
1818181888	8111818111	1888111181	8181888118	1818181188	8111818811	1888111881	8181888181	1818181811	8111818188	1888111818
1888181111	8181881888	1818188818	8111811881	1888118811	8181881188	1818188118	8111811818	1888118188	8181881811	1818188181
13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986
10	11	12	13	14	15	16	17	18	19	20



13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986
1188881181	8811181881	1118818118	8881118818	1188881118	8811181818	1118818181	8881118881	13999999986
1118818818	8881118118	1188881881	8811181181	1118818881	8881118181	1188881818	8811181118	13999999986
8881181881	1118881181	8811118818	1188818118	8881181818	1118881118	8811118881	1188818181	13999999986
8811118118	1188818818	8881181181	1118881881	8811118181	1188818881	8881181118	1118881818	13999999986
1888881181	8111181881	1111818118	8888118818	1888881118	8111181818	1111818181	8888118881	13999999986
1111818818	8888118118	1888881881	8111181181	1111818881	8888118181	1888881818	8111181118	13999999986
8888181881	1111881181	8111118818	1888818118	8888181818	1111881118	8111118881	1888818181	13999999986
8111118118	1888818818	8888181181	1111881881	8111118181	1888818881	8888181118	1111881818	13999999986
1811181181	8188881881	1118118118	8881818818	1811181118	8188881818	1118118181	8881818881	13999999986
1118118818	8881818118	1811181881	8188881181	1118118881	8881818181	1811181818	8188881118	13999999986
8881881881	1118181181	8188818818	1811118118	8881881818	1118181118	8188818881	1811118181	13999999986
8188818118	1811118818	8881881181	1118181881	8188818181	1811118881	8881881118	1118181818	13999999986
1881181181	8118881881	1188118118	8811818818	1881181118	8118881818	1188118181	8811818881	13999999986
1188118818	8811818118	1881181881	8118881181	1188118881	8811818181	1881181818	8118881118	13999999986
8811881881	1188181181	8118818818	1881118118	8811881818	1188181118	8118818881	1881118181	13999999986
8118818118	1881118818	8811881181	1188181881	8118818181	1881118881	8811881118	1188181818	13999999986
1818881181	8181181881	1181818118	8818118818	1818881118	8181181818	1181818181	8818118881	13999999986
1181818818	8818118118	1818881881	8181181181	1181818881	8818118181	1818881818	8181181118	13999999986
8818181881	1181881181	8181118818	1818818118	8818181818	1181881118	8181118881	1818818181	13999999986
8181118118	1818818818	8818181181	1181881881	8181118181	1818818881	8818181118	1181881818	13999999986
1881881181	8118181881	1811818118	8188118818	1881881118	8118181818	1811818181	8188118881	13999999986
1811818818	8188118118	1881881881	8118181181	1811818881	8188118181	1881881818	8118181118	13999999986
8188181881	1811881181	8118118818	1881818118	8188181818	1811881118	8118118881	1881818181	13999999986
8118118118	1881818818	8188181181	1811881881	8118118181	1881818881	8188181118	1811881818	13999999986
1888181181	8111881881	1818118118	8181818818	1888181118	8111881818	1818118181	8181818881	13999999986
1818118818	8181818118	1888181881	8111881181	1818118881	8181818181	1888181818	8111881118	13999999986
8181881881	1818181181	8111818818	1888118118	8181881818	1818181118	8111818881	1888118181	13999999986
8111818118	1888118818	8181881181	1818181881	8111818181	1888118881	8181881118	1818181818	13999999986
13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986	13999999986
21	22	23	24	25	26	27	28	

It is based on 2 by 2 combinations of 28 numbers:

$$\{11118, 11181, 11188, 11818, 11881, 11888, 18111, 18118, 18181, 18188, 18811, 18818, 18881, 18888, 81111, 81118, 81181, 81188, 81811, 81818, 81881, 81888, 88111, 88118, 88181, 88811, 88818, 88881\}.$$

These 28 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{18888, 88881\}$ ,  $\{11188, 88111\}$ ,  $\{11888, 88811\}$ ,  $\{11881, 18811\}$ ,  $\{81188, 88118\}$ ,  $\{11181, 18111\}$ ,  $\{81888, 88818\}$ ,  $\{11818, 81811\}$ ,  $\{18188, 88181\}$ ,



$\{18118, 81181\}$  and  $\{18818, 81881\}$ . The numbers  $\{18181, 18881, 81118, 81818\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.11 is **universal magic square**. Its **magic sum** is

$$S_{28 \times 28} := 139999999986 = 100000 \times 1399986 + 1399986,$$

where

$$\begin{aligned} 1399986 := & 11118 + 11181 + 11188 + 11818 + 11881 + 11888 + 18111 + 18118 + 18181 + 18188 + \\ & + 18811 + 18818 + 18881 + 18888 + 81111 + 81118 + 81181 + 81188 + 81811 + \\ & + 81818 + 81881 + 81888 + 88111 + 88118 + 88181 + 88811 + 88818 + 88881 \end{aligned}$$

The blocks of order 28 appearing in Example 2.20 are of equal magic sums. The blocks are constructed dividing the above 28 numbers in seven parts with equal sums:

$$A := 11118 + 18888 + 81111 + 88881 = 199998$$

$$B := 11188 + 11888 + 88111 + 88811 = 199998$$

$$C := 11881 + 18811 + 81188 + 88118 = 199998$$

$$D := 11181 + 18111 + 81888 + 88818 = 199998$$

$$E := 18181 + 18881 + 81118 + 81818 = 199998$$

$$F := 11818 + 18188 + 81811 + 88181 = 199998$$

$$G := 18118 + 18818 + 81181 + 81881 = 199998$$

Based on these seven groups  $A, B, C, D, E, F$  and  $G$ , 28 magic squares of order 4 are constructed with equal **magic sums**  $S_{4 \times 4} := 1999999998$ . These 28 magic squares lead us to a **pandiagonal magic square** of order 28 given in Example 2.20.

### 2.12.2 Blocks of Order 7

**Example 2.21.** *The pandiagonal magic square of order 28 for the digits 1 and 8 is given below in three parts:*

28x28	mgc								
8888188881	1188811888	1188111881	1811118111	8181881818	1181811818	8188181881	8111111118	8881188111	8118888118
1181881818	8188111818	8888181881	1188888881	1188111888	1811111881	8181818111	1818818181	1881881811	8111118118
1811111888	8181811881	1181818111	8188181818	8888111818	1188881881	1188188881	1118188111	1888188118	1818881888
1188811818	1188181881	1811188881	8181811888	1181811881	8188118111	8888181818	8881181811	8118818118	1118111118
8188111881	8888118111	1188881818	1188111818	1811181881	8181888881	1181811888	1881888118	8111181888	8881118181
8181881881	1181888881	8188111888	8888111881	1188818111	1188181818	1811111818	1888118118	1818811118	1881888111
1188118111	1811181818	8181811818	1181881881	8188188881	8888111888	1188811881	8118881888	1118118181	1888181811
1111881111	8811188811	8811881188	8188811181	1818118881	8181118188	1811818818	1888818888	1118811188	1881118811
8181118881	1811818188	1111818818	8811181111	8811888811	8188881188	1818111181	8818181118	8118188181	1888881181
8188888811	1818181188	8181111181	1811818881	1111818188	8811118818	8811881111	8881811188	8111818811	8818188818
8811118188	8811818818	8188881111	1818188811	8181181188	1811811181	1111818881	1118888181	1881181181	8881818888
1811881188	1111811181	8811118881	8811818188	8188818818	1818181111	8181188811	8118118811	1888888818	1118881118
1818118818	8181181111	1811888811	1111881188	8811111181	8811818881	8188818188	8111881181	8818118888	8118111188
8811811181	8188818881	1818118188	8181118818	1811881111	1111888811	8811181188	1881188818	8881881118	8111888181
1888811118	1118888111	1881188118	8881881888	8111818181	8818181811	8118118118	1111888881	8811111888	8811811881
8818118181	8118181811	1888818118	1118811118	1881188111	8881888118	8111881888	8181181818	1811811818	1111881881
8881888111	8111888118	8818181888	8118118181	1888881811	1118818118	1881111118	8188811888	1818111881	8181118111
1118881811	1881118118	8881811118	8111888111	8818188118	8118181888	1888818181	8811111818	8811881881	8188888881
8118188118	1888881888	1118818181	1881181811	8881818118	8111811118	8818188111	1811811881	1111818111	8811181818
8111818118	8818111118	8118188111	1888888118	1118881888	1881118181	8881881811	1818181881	8181188881	1811811888
1881181888	8881818181	8111881811	8818118118	8118111118	1888888111	1118888118	8811818111	8188881818	1818111818
8111118888	8881111188	8118818811	1118188818	1888181118	1818888181	1881881181	8888181111	1188888811	1188181188
1818881118	1881888181	8111181181	8881118888	8118811188	1118118811	1888188818	1181818881	8188118188	8888118818
1118111188	1888118811	1818888818	1881881118	8111188181	8881181181	8118818888	1811188811	8181881188	1181811181
8881188181	8118881181	1118118888	1888111188	1818818811	1881888818	8111181118	1188818188	1188118818	1811181111
1881818811	8111188818	8881181118	8118888181	1118181181	1888118888	1818811188	8188181188	8888111181	1188818881
1888181181	1818818888	1881811188	8111118811	8881188818	8118881118	1118188181	8181818818	1181881111	8188188811
8118888818	1118181118	1888188181	1818881181	1881818888	8111111188	8881118811	1188111181	1811118881	8181818188
139999999986	139999999986	139999999986	139999999986	139999999986	139999999986	139999999986	139999999986	139999999986	139999999986
1	2	3	4	5	6	7	8	9	10



1118181888	1888118181	1818881811	1881818118	1111818888	8811111188	8811818811	8188888818	1818181118	8181188181
8881111118	8118888111	1118188118	1888181888	8181181118	1811888181	1111881181	8811118888	8811811188	8188818811
1881818181	8111181811	8881118118	8118811118	8188811188	1818118811	8181188818	1811881118	1111888181	8811181181
1888188111	1818888118	1881881888	8111118181	8811188181	8811881181	8188818888	1818111188	8181118811	1811888818
8118881811	1118118118	1888111118	1818888111	1811818811	1111888818	8811181118	8811888181	8188881181	1818118888
8111188118	8881181888	8118818181	1118181811	1818181181	8181118888	1811811188	1111818811	8811188818	8811881118
1818818118	1881811118	8111188111	8881188118	8811888818	8188881118	1818188181	8181181181	1811818888	1111811188
8881888818	8111881118	8818188181	8118181181	8888111118	1188888111	1188188118	1811181888	8181818181	1181881811
1118818888	1881111188	8881818811	8111888818	1181818181	8188181811	8888118118	1188811118	1188188111	1811188118
8118181118	1888888181	1118881181	1881118888	1811188111	8181888118	1181881888	8188118181	8888181811	1188818118
8111811188	8818118811	8118188818	1888881118	1188881811	1188118118	1811111118	8181888111	1181888118	8188181888
1881188181	8881881181	8111818888	8818111188	8188188118	8888181888	1188818181	1188181811	1811118118	8181811118
1888818811	1118888818	1881181118	8881888181	8181818118	1181811118	8188188111	8888188118	1188881888	1188118181
8818181181	8118118888	1888811188	1118818811	1188181888	1811118181	8181881811	1181818118	8188111118	8888188111
8188818111	1818181818	8181111818	1811881881	8111181111	8881188811	8118881188	1118111181	1888118881	1818818188
8811188881	8811811888	8188811881	1818118111	1818818881	1881818188	8111118818	8881181111	8118888811	1118181188
1811881818	1111811818	8811181881	8811888881	1118188811	1888181188	1818811181	1881818881	8111118188	8881118818
1818111888	8181111881	1811818111	1111881818	8881118188	8118818818	1118181111	1888188811	1818881188	1881811181
8811811818	8188881881	1818188881	8181111888	1881881188	8111111181	8881118881	8118818188	1118118818	1888181111
1111811881	8811118111	8811881818	8188811818	1888118818	1818881111	1881888811	8111181188	8881111181	8118818881
8181181881	1811888881	1111811888	8811111881	8118811181	1118118881	1888118188	1818818818	1881881111	8111188811
1811111181	8181818881	1181818188	8188118818	1888888881	1118811888	1881111881	8881818111	8111881818	8818111818
1188881111	1188188811	1811181188	8181811181	8818181818	8118111818	1888881881	1118888881	1881111888	8881811881
8188118881	8888118188	1188818818	1188181111	8881811888	8111811881	8818118111	8118181818	1888811818	1118881881
8181888811	1181881188	8188111181	8888118881	1118811818	1881181881	8881888881	8111811888	8818111881	8118118111
1188118188	1811118818	8181881111	1181888811	8118111881	1888818111	1118881818	1881111818	8881881881	8111888881
8888181188	1188811181	1188118881	1811118188	8111881881	8818188881	8118111888	1888811881	1118818111	1881181818
1181818818	8188181111	8888188811	1188881188	1881118111	8881881818	8111811818	8818181881	8118188881	1888811888
139999999986	139999999986	139999999986	139999999986	139999999986	139999999986	139999999986	139999999986	139999999986	139999999986
11	12	13	14	15	16	17	18	19	20



								139999999986
1811881181	1888881111	1118888811	1881181188	8881811181	8111818881	8818118188	8118118818	139999999986
1818188818	8818118881	8118118188	1888818818	1118881111	1881188811	8881881188	8111811181	139999999986
8811818888	8881888811	8111881188	8818111181	8118118881	1888818188	1118818818	1881181111	139999999986
1111881118	1118818188	1881118818	8881881111	8111888811	8818181188	8118111181	1888818881	139999999986
8181111188	8118181188	1888811181	1118818881	1881118188	8881818818	8111881111	8818188811	139999999986
8188888181	8111818818	8818181111	8118188811	1888881188	1118811181	1881118881	8881818188	139999999986
8811118811	1881111181	8881818881	8111818188	8818118818	8118181111	1888888811	1118881188	139999999986
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8181881888	1818881818	1881811818	8111181881	8881188881	8118811888	1118111881	1888118111	139999999986
1188111118	1118111888	1888111881	1818818111	1881881818	8111111818	8881181881	8118888881	139999999986
8888118181	8881111818	8118881881	1118188881	1888111888	1818811881	1881818111	8111181818	139999999986
1181888111	1881811881	8111118111	8881181818	8118811818	1118181881	1888188881	1818811888	139999999986
1811181811	1888181881	1818888881	1881811888	8111111881	8881118111	8118881818	1118111818	139999999986
1188888118	8118818111	1118181818	1888111818	1818881881	1881888881	8111111888	8881111881	139999999986
1881818818	8888118888	1188811188	1188118811	1811188818	8181881118	1181888181	8188181181	139999999986
1888111181	1181881118	8188188181	8888181181	1188818888	1188111188	1811118811	8181888818	139999999986
8118881111	1811111188	8181818811	1181888818	8188181118	8888188181	1188881181	1188118888	139999999986
8111118881	1188888181	1188181181	1811118888	8181811188	1181818811	8188188818	8888181118	139999999986
1818888811	8188118811	8888188818	1188881118	1188188181	1811181181	8181818888	1181811188	139999999986
1118118188	8181881181	1181818888	8188111188	8888118811	1188888818	1188181118	1811188181	139999999986
8881181188	1188188818	1811181118	8181888181	1181881181	8188118888	8888111188	1188818811	139999999986
8118181881	1111811118	8811188111	8811888118	8188881888	1818118181	8181181811	1811818118	139999999986
8111818111	8181118181	1811881811	1111818118	8811111118	8811888111	8188888118	1818181888	139999999986
1881188881	8188888111	1818188118	8181181888	1811818181	1111881811	8811118118	8811811118	139999999986
1888881818	8811181811	8811818118	8188811118	1818188111	8181188118	1811881888	1111818181	139999999986
8818111888	1811888118	1111881888	8811118181	8811881811	8188818118	1818111118	8181188111	139999999986
8881811818	1818118118	8181111118	1811888111	1111888118	8811181888	8811818181	8188881811	139999999986
1118811881	8811881888	8188818181	1818181811	8181118118	1811811118	1111888111	8811188118	139999999986
139999999986	139999999986	139999999986	139999999986	139999999986	139999999986	139999999986	139999999986	139999999986
21	22	23	24	25	26	27	28	

This magic square is also constructed with the same number as of Example 2.20. These numbers lead us to a **universal magic square** with **magic sum**, i.e.,

$$S_{28 \times 28} := 139999999986 = 100000 \times 1399986 + 1399986,$$

The magic square given in Example 2.21 is constructed based on blocks of order 7. These blocks are **pandiagonal** magic squares of order 7 with different magic sums resulting in a **magic square** of order 28.



2.13 Magic Square of Order 29

**Example 2.22.** *The pandiagonal magic square of order 29 for the digits 1 and 8 is given below in three parts:*

29x29	pan	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174
	1888118881	8818811188	1111111888	8888811118	8811818888	8118811181	8118188818	1811811818	8188118188
148818888174	1118888188	8111881118	1888188111	8818888811	1111181111	8888888881	8811818111	8118881888	8118181811
148818888174	1188811111	8811118881	1118811188	8111811888	1888111118	8818818888	1111111811	8888888818	8811811818
148818888174	1111888888	8881188188	1188881118	8811188111	1118888811	8111881111	1888188881	8818818111	1111181888
148818888174	1888888118	8111111111	1111818881	8881111188	1188811888	8811111118	1118818888	8111811181	1888188818
148818888174	1118181188	8888188888	1888888188	8111181118	1111888111	8881188811	1188881111	8811188881	1118818111
148818888174	8881881181	1811188118	1118111111	8888118881	1888811188	8111111888	1111811118	8881118888	1188811181
148818888174	1181818118	8188881188	8881888888	1811188188	1118181118	8888188111	1888888111	8111181111	1111888881
148818888174	1818881881	8181181181	1181888118	8188811111	8881818881	1811111188	1118111888	8888111118	1888818888
148818888174	1188118818	8818118118	1818881188	8181188888	1181888188	8188881118	8881888111	1811188811	1118181111
148818888174	1881818811	1881181881	1188181181	8818188118	1818811111	8181118881	1181811188	8188811888	8881811118
148818888174	1811811881	8188118818	1881818118	1881181188	1188188888	8818188188	1818881118	8181188111	1181888811
148818888174	8118888181	8118118811	1811881881	8188181181	1881888118	1881111111	1188118881	8818111188	1818811888
148818888174	8888818188	8811811881	8118818818	8118118118	1811881188	8188188888	1881888188	1881181118	1188188111
148818888174	8818881811	1111188181	8888818811	8811881881	8118881181	8118188118	1811811111	8188118881	1881811188
148818888174	8111811818	1888118188	8818811881	1111118818	8888818118	8811881188	8118888888	8118188188	1811881118
148818888174	8811181888	1118881811	8111888181	1888118811	8818881881	1111181181	8888888118	8811811111	8118818881
148818888174	8881188818	1188811818	8811118188	1118811881	8111818818	1888118118	8818881188	1111188888	8888888188
148818888174	8111118111	1111881888	8881181811	1188888181	8811118811	1118881881	8111881181	1888188118	8818811111
148818888174	8888111181	1888888188	8111111818	1111818188	8881111881	1188818818	8811118118	1118881188	8111888888
148818888174	1811188881	1118118111	8888181888	1888881811	8111188181	1111818811	8881181881	1188881181	8811188118
148818888174	8188818888	8881811181	1811188818	1118111818	8888118188	1888811881	8111118818	1111818118	8881181188
148818888174	8181181111	1181888881	8188818111	8881881888	1811181811	1118188181	8888118811	1888881881	8111181181
148818888174	8818111118	1818818888	8181111181	1181888818	8188811818	8881818188	1811111881	1118118818	8888118118
148818888174	1881188811	1188181111	8818188881	1818818111	8181181888	1181881811	8188888181	8881818811	1811181881
148818888174	8188111888	1881811118	1881118888	1188111181	8818188818	1818811818	8181118188	1181811881	8188818818
148818888174	8118188111	1811888811	8188181111	1881888881	1881118111	1188181888	8818181811	1818888181	8181118811
148818888174	8811811188	8118811888	8118111118	1811818888	8188111181	1881888818	1881111818	1188118188	8818111881
148818888174	1111181118	8888888111	8811888811	8118881111	8118188881	1811818111	8188181888	1881881811	1881188181
	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174
	1	2	3	4	5	6	7	8	9



148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174
1881811881	1881118818	1188118118	8818181188	1818888888	8181188188	1181881118	8188888111	8881888811	1811181111	1118188881
1811888181	8188118811	1881881881	1881181181	1188188118	8818111111	1818818881	8181111188	1181811888	8188811118	8881818888
8118818188	8118111881	1811818818	8188118118	1881881188	1881188888	1188188188	8818181118	1818888111	8181188811	1181881111
8888881811	8811888181	8118818811	8118181881	1811881181	8188188118	1881811111	1881118881	1188111188	8818111888	1818811118
8818811818	1111118188	8888811881	8811818818	8118818118	8118181188	1811888888	8188188188	1881881118	1881188111	1188188811
8111881888	1888181811	8818888181	1111118811	8888881881	8811881181	8118888118	8118111111	1811818881	8188111188	1881811888
8811188818	1118811818	8111818188	1888111881	8818818818	1111118118	8888881188	8811888888	8118888188	8118181118	1811888111
8881118111	1188881888	8811181811	1118888181	8111818811	1888181881	8818881181	1111188118	8888811111	8811818881	8118811188
8111111181	1111888818	8881111818	1188818188	8811111881	1118818818	8111818118	1888181188	8818888888	1111188188	8888881118
8888188881	1888818111	8111181888	1111881811	8881188181	1188818811	8811181881	1118881181	8111888118	1888111111	8818818881
1811118888	1118111181	8888188818	1888811818	8111118188	1111811881	8881118818	1188818118	8811181188	1118888888	8111888188
8188881111	8881888881	1811118111	1118181888	8888181811	1888888181	8111118811	1111881881	8881181181	1188888118	8811111111
8181111118	1181818888	8188811181	8881888818	1811111818	1118118188	8888111881	1888818818	8111118118	1111881188	8881188888
8818188811	1818881111	8181188881	1181818111	8188881888	8881881811	1811188181	1118118811	8888181881	1888881181	8111188118
1881111888	1188111118	8818118888	1818811181	8181188818	1181811818	8188818188	8881811881	1811118818	1118118118	8888181188
8188188111	1881888811	1881181111	1188188881	8818118111	1818881888	8181181811	1181888181	8188818811	8881881881	1811181181
8118111188	1811811888	8188111118	1881818888	1881111181	1188188818	8818111818	1818818188	8181111881	1181818818	8188818118
8811881118	8118888111	8118188811	1811881111	8188188881	1881818111	1881181888	1188181811	8818188181	1818818811	8181181881
1111118881	8888811188	8811811888	8118811118	8118118888	1811811181	8188188818	1881811818	1881118188	1188111881	8818118818
1888188188	8818881118	1111188111	8888888811	8811881111	8118888881	8118118111	1811881888	8188181811	1881888181	1881118811
1118811111	8111818881	1888111188	8818811888	1111111118	8888818888	8811811181	8118888818	8118111818	1811818188	8188111881
1188888888	8811188188	1118881118	8111888111	1888188811	8818881111	1111188881	8888818111	8811881888	8118881811	8118188181
1111888118	8881111111	1188818881	8811111188	1118811888	8111811118	1888118888	8818811181	1111188818	8888811818	8811818188
1888881188	8111188888	1111888188	8881181118	1188888111	8811188811	1118881111	8111888881	1888118111	8818881888	1111181811
1118181181	8888188118	1888811111	8111118881	1111811188	8881111888	1188811118	8811118888	1118811181	8111888818	1888111818
8881818118	1811181188	1118188888	8888188188	1888881118	8111188111	1111888811	8881181111	1188888881	8811118111	1118881888
1181881881	8188881181	8881888118	1811111111	1118118881	8888111188	1888811888	8111111118	1111818888	8881111181	1188888818
1818818818	8181118118	1181881188	8188888888	8881888188	1811181118	1118188111	8888188811	1888881111	8111188881	1111818111
1188118811	8818181881	1818881181	8181188118	1181811111	8188818881	8881811188	1811111888	1118111118	8888118888	1888811181
148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174
10	11	12	13	14	15	16	17	18	19	20



148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174
1888881888	8111181811	1111888181	8881118811	1188881881	8811181181	1118888118	8111811111	148818888174
1118188818	8888111818	1888818188	8111111881	1111818818	8881118118	1188881188	8811188888	148818888174
8881818111	1811181888	1118181811	8888188181	1888818811	8111181881	1111881181	8881188118	148818888174
1181811181	8188888818	8881811818	1811118188	1118111881	8888118818	1888818118	8111181188	148818888174
1818888881	8181118111	1181881888	8188881811	8881888181	1811118811	1118181881	8888181181	148818888174
1188118888	8818111181	1818888818	8181111818	1181818188	8188811881	8881818818	1811118118	148818888174
1881881111	1881188881	1188118111	8818181888	1818881811	8181188181	1181818811	8188881881	148818888174
1811811118	8188118888	1881811181	1881188818	1188111818	8818118188	1818811881	8181118818	148818888174
8118888811	8118181111	1811888881	8188118111	1881881888	1881181811	1188188181	8818118811	148818888174
8888811888	8811811118	8118818888	8118111181	1811888818	8188111818	1881818188	1881111881	148818888174
8818888111	1111188811	8888881111	8811888881	8118818111	8118181888	1811881811	8188188181	148818888174
8111811188	1888111888	8818811118	1111118888	8888811181	8811888818	8118811818	8118118188	148818888174
8811181118	1118888111	8111888811	1888181111	8818888881	1111118111	8888881888	8811881811	148818888174
8881118881	1188811188	8811111888	1118811118	8111818888	1888111181	8818888818	1111111818	148818888174
8111188188	1111881118	8881188111	1188888811	8811181111	1118888881	8111818111	1888181888	148818888174
8888111111	1888818881	8111111188	1111811888	8881111118	1188818888	8811111181	1118888818	148818888174
1811188888	1118188188	8888181118	1888888111	8111188811	1111881111	8881188881	1188818111	148818888174
8188888118	8881811111	1811118881	1118111188	8888111888	1888811118	8111118888	1111811181	148818888174
8181181188	1181888888	8188888188	8881881118	1811188111	1118188811	8888181111	1888888881	148818888174
8818181181	1818888118	8181111111	1181818881	8188811188	8881811888	1811111118	1118118888	148818888174
1881118118	1188181188	8818188888	1818888188	8181181118	1181888111	8188888811	8881881111	148818888174
8188181881	1881881181	1881188118	1188111111	8818118881	1818811188	8181111888	1181811118	148818888174
8118118818	1811818118	8188181188	1881888888	1881188188	1188181118	8818188111	1818888811	148818888174
8811818811	8118881881	8118181181	1811888118	8188111111	1881818881	1881111188	1188111888	148818888174
1111111881	8888818818	8811818118	8118881188	8118188888	1811888188	8188181118	1881888111	148818888174
1888188181	8818818811	1111181881	8888881181	8811888118	8118811111	8118118881	1811811188	148818888174
1118818188	8111811881	1888118818	8818818118	1111181188	8888888888	8811888188	8118881118	148818888174
1188881811	8811188181	1118818811	8111881881	1888181181	8818888118	1111111111	8888818881	148818888174
1111811818	8881118188	1188811881	8811118818	1118818118	8111881188	1888188888	8818888188	148818888174
148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174	148818888174
22	23	24	25	26	27	28	29	

It is based on 2 by 2 combinations of 29 numbers:

{11111, 11118, 11181, 11188, 11818, 11881, 11888, 18111, 18118, 18188, 18811, 18818, 18881, 18888, 81111, 81118, 81181, 81188, 81811, 81881, 81888, 88111, 88118, 88181, 88188, 88811, 88818, 88881, 88888}.

These 29 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{11181, 18111\}$ ,  $\{11188, 88111\}$ ,  $\{11818, 81811\}$ ,  $\{11888, 88811\}$ ,  $\{11881, 18811\}$ ,  $\{18118, 81181\}$ ,  $\{18188, 88181\}$ ,  $\{18818, 81881\}$ ,  $\{18888, 88881\}$ ,  $\{81188, 88118\}$  and  $\{81888, 88818\}$ . The numbers  $\{11111, 18881, 81118, 88188, 88888\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.22 is **universal pan-diagonal magic square**. Its **magic sum** is

$$S_{29 \times 29} := 148818888174 = 100000 \times 1488174 + 1488174$$

where

$$\begin{aligned} 1488174 := & 11111 + 11118 + 11181 + 11188 + 11818 + 11881 + 11888 + 18111 + 18118 + 18188 + \\ & + 18811 + 18818 + 18881 + 18888 + 81111 + 81118 + 81181 + 81188 + 81811 + 81881 + \\ & + 81888 + 88111 + 88118 + 88181 + 88188 + 88811 + 88818 + 88881 + 88888. \end{aligned}$$

## 2.14 Magic Square of Order 30

### 2.14.1 Blocks of Order 6

**Example 2.23.** *The magic square of order 30 for the digits 1 and 8 is given below in three parts:*



30x30	mgc								
1118111181	8881888811	8881888111	8881811888	1118111188	1118188818	1811111118	8188888188	8188881111	8188818888
8881188818	1118811188	8881188111	1118811888	1118888811	8881111181	8188188881	1811811811	8188181111	1811818888
8811188818	8811188811	1188811888	1188888111	8811111188	1188811181	8118188881	8118188188	1881818888	1881881111
1188888818	1188811188	8811111888	8811188111	1188888811	8811111181	1881888881	1881811811	8118118888	8118181111
1118811181	8881111188	1118888111	8881111888	8881188811	1118888818	1811811118	8188111811	1811881111	8188118888
8881811181	1118188811	1118111888	1118188111	8881811188	8881888818	8188811118	1811188188	1811118888	1811181111
1111818111	8888181881	8888181181	8888118818	1111818118	1111881888	1181811818	8818181818	8818181811	8818118188
8818881888	1181118118	8818881181	1181118818	1181181881	8818818111	8181888181	1818118181	8181881811	1818118188
8111181888	8111181881	1888818818	1888881181	8111118118	1888818111	8181188181	8181181818	1818818188	1818881811
1888881888	1888818118	8111118818	8111181181	1888881881	8111118111	1818888181	1818818181	8181118188	8181181811
1181118111	8818818118	1181181181	8818818818	8818881881	1181181888	1818111818	8181818181	1818181811	8181818188
8888118111	1111881881	1111818818	1111881181	8888118118	8888181888	8818111818	1181881818	1181818188	1181881811
1811111111	8188888118	8188881188	8188818811	1811111881	1811188888	1111111181	8888888811	8888888111	8888811888
8188188888	1811811881	8188181188	1811818811	1811888118	8188111111	8811888818	1188111188	8811888111	1188111888
8118188888	8118188118	1881818811	1881881188	8118111881	1881811111	8118888818	8118888811	1881111888	1881188111
1881888888	1881811881	8118118811	8118181188	1881888118	8118111111	1881188818	1881111188	8118811888	8118888111
1811811111	8188111881	1811881188	8188118811	8188188118	1811888888	1188111181	8811811188	1188188111	8811811888
8188811111	1811188118	1811118811	1811181188	8188811881	8188888888	8888811181	1111188811	1111111888	1111188111
1181811118	8818188188	8818181111	8818118888	1181811811	1181888881	1118118111	8881881881	8881881181	8881818818
8181888881	1818111811	8181881111	1818118888	1818188188	8181811118	8881181888	1118818118	8881181181	1118818818
8181188881	8181188188	1818818888	1818881111	8181111811	1818811118	8811181888	8811181881	1188818818	1188881181
1818888881	1818811811	8181118888	8181181111	1818888188	8181111118	1188881888	1188818118	8811118818	8811181181
1818111118	8181811811	1818181111	8181818888	8181888188	1818188881	1118818111	8881118118	1118881181	8881118818
8818111118	1181888188	1181818888	1181881111	8818111811	8818188881	8881818111	1118181881	1118118818	1118181181
1111111181	8888888811	8888888111	8888811888	1111111188	1111188818	1111811111	8888188118	8888181188	8888118811
8811888818	1188111188	8811888111	1188111888	1188188811	8811811181	8818888888	1181111881	8818881188	1181118811
8118888818	8118888811	1881111888	1881188111	8118811188	1881111181	8111188888	8111188118	1888818811	1888881188
1881188818	1881111188	8118811888	8118888111	1881188811	8118811181	1888888888	1888811881	8111118811	8111181188
1188111181	8811811188	1188188111	8811811888	8811888811	1188188818	1181111111	8818811881	1181181188	8818818811
8888811181	1111188811	1111111888	1111188111	8888811188	8888888818	8888111111	1111888118	1111818811	1111881188
14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985
1	2	3	4	5	6	7	8	9	10

181111811	181188881	111118111	8888881881	8888881181	8888818818	111118118	111181888	1111811818	8888181818
181888188	818811118	881881888	118818118	881881181	118818818	1188181881	881818111	8818888181	118118181
818111811	188181118	818881888	818881881	188118818	188181181	818818118	188118111	811188181	811181818
1881888188	818111118	188181888	188118118	818818818	818881181	188181881	818818111	1888888181	1888818181
8188188188	181888881	118818111	881818118	1188181181	881818818	881881881	1188181888	118111818	8818818181
8188811811	8188888881	8888818111	1111181881	111118818	111181181	8888818118	8888881888	888811818	1111881818
1181818181	1181888181	1118111111	8881888118	8881881188	8881818811	1118111881	1118188888	181111181	8188888811
1818181818	8181811818	888188888	1118811881	888181188	1118818811	1118888118	8881111111	8188188818	181811188
818118181	1818811818	8811188888	8811188118	1188818811	1188881188	8811111881	1188811111	8118188818	8118188811
1818881818	818111818	1188888888	1188811881	8811118811	8811181188	1188888118	8811111111	1881888818	1881811188
8181881818	1818188181	1118811111	8881111881	1118881188	8881118811	888188118	1118888888	181811181	8188111188
8818118181	8818188181	8881811111	1118188118	1118118811	1118181188	8881811881	8881888888	8188811181	1811188811
1111111188	111188818	1111811118	8888188188	8888181111	8888118888	1111811811	1111888881	1181818111	8818181881
1188188811	881811181	8818888881	1181111811	8818881111	1181118888	1181188188	8818811118	8181881888	181818118
8118811188	1881111181	8111188881	8111188188	1888818888	1888881111	811111811	1888811118	818181888	8181181881
1881188811	8118811181	1888888881	1888811811	8111118888	8111181111	1888888188	8111111118	1818881888	1818818118
8811888811	1188188818	1181111118	8818811811	1181811111	8818818888	8818888188	1181188881	1818181111	8181818118
8888811188	8888888818	8888111118	1111888188	1111818888	1111881111	8888111811	8888188881	8818118111	1181881881
111818118	1118181888	1811111818	8188881818	8188881811	8188818188	1811118181	1811188181	1111111111	8888888118
1118881881	8881118111	8188188181	1811818181	8188181811	1811818188	1811881818	8188111818	8811888888	1188111881
8811118118	1188818111	8118188181	8118181818	1881818188	1881881811	8118118181	1881811818	8118888888	8118888118
1188881881	8811118111	1881888181	1881818181	8118118188	8118181811	1881881818	8118111818	1881188888	1881111881
8881181881	1118881888	1811811818	8188118181	1811881811	8188118188	8188181818	1811888181	1188111111	8811811881
8881818118	8881881888	8188811818	1811181818	1811118188	1811181811	8188818181	8188888181	8888811111	1111188118
1111811881	1111888888	1181811181	8818188811	8818188111	8818111888	1181811188	1181888818	1118111118	8881888188
1181188118	8818811111	8181888818	1818111188	8181888111	1818111888	1818188811	8181811181	8881188881	1118811811
8111111881	1888811111	8181188818	8181188811	1818811888	1818888111	8181111188	1818811181	8811188881	8811188188
1888888118	8111111111	1818888818	1818811188	8181111888	8181188111	1818888811	8181111181	1188888881	1188811811
8818888118	1181188888	1818111181	8181811188	1818188111	8181811888	8181888811	1818188818	1118811118	8881111811
8888111881	8888188888	8818111181	1181888811	1181811888	1181888111	8818111188	8818188818	8881811118	1118188188
14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985
11	12	13	14	15	16	17	18	19	20



										4999999985
8888181811	8888118188	1111818181	1111888181	1181811111	8818188118	8818181188	8818118811	1181811881	1181888888	4999999985
8818881811	1181118188	1181181818	8818811818	8181888888	1818111881	8181881188	1818118811	1818188118	8181811111	4999999985
1888818188	1888881811	8111118181	1888811818	8181188888	8181188118	1818818811	1818881188	8181111881	1818811111	4999999985
8111118188	8111181811	1888881818	8111111818	1818888888	1818811881	8181118811	8181181188	1818888118	8181111111	4999999985
1181181811	8818818188	8818881818	1181188181	1818111111	8181811881	1818181188	8181818811	8181888118	1818188888	4999999985
1111818188	1111881811	8888118181	8888188181	8818111111	1181888118	1181818811	1181881188	8818111881	8818188888	4999999985
8188888111	8188811888	1811111188	1811188818	1111111118	8888888188	8888881111	8888818888	1111111811	1111188881	4999999985
8188188111	1811811888	1811888811	8188111181	8811888881	1188111811	8811881111	1188118888	1188188188	8811811118	4999999985
1881811888	1881888111	8118111188	1881811181	8118888881	8118888188	1881118888	1881181111	8118811811	1881111118	4999999985
8118111888	8118188111	1881888811	8118111181	1881188881	1881111811	8118818888	8118881111	1881188188	8118811118	4999999985
1811888111	8188111888	8188188811	1811888818	1188111118	8811811811	1188181111	8811818888	8811888188	1188188881	4999999985
1811111888	1811188111	8188811188	8188888818	8888811118	1111188188	1111118888	1111181111	8888811811	8888888881	4999999985
8818181181	8818118818	1181818118	1181881888	1118111818	8881881818	8881881811	8881818188	1118118181	1118188181	4999999985
8181881181	1818118818	1818181881	8181818111	8881188181	1118818181	8881181811	1118818188	1118881818	8881111818	4999999985
1818818818	1818881181	8181118118	1818818111	8811188181	8811181818	1188818188	1188881811	8811118181	1188811818	4999999985
8181118818	8181181181	1818881881	8181118111	1188888181	1188818181	8811118188	8811181811	1188881818	8811111818	4999999985
1818181181	8181818818	8181881881	1818181888	1118811818	8881118181	1118881811	8881118188	8881181818	1118888181	4999999985
1181818818	1181881181	8818118118	8818181888	8881811818	1118181818	1118118188	1118181811	8881818181	8881888181	4999999985
8888881188	8888818811	1111111881	1111188888	1111811181	8888188811	8888188111	8888111888	1111811188	1111888818	4999999985
8811881188	1188118811	1188188118	8811811111	8818888818	1181111188	8818888111	1181111888	1181188811	8818811181	4999999985
1881118811	1881181188	8118811881	1881111111	8111188818	8111188811	1888811888	1888888111	8111111188	1888811181	4999999985
8118818811	8118881188	1881188118	8118811111	1888888818	1888811188	8111111888	8111188111	1888888811	8111111181	4999999985
1188181188	8811818811	8811888118	1188188888	1181111181	8818811188	1181188111	8818811888	8818888811	1181188818	4999999985
1111118811	1111181188	8888811881	8888888888	8888111181	1111888811	1111811888	1111888111	8888111188	8888188818	4999999985
8881881111	8881818888	1118111811	1118188881	1811118111	8188881881	8188881181	8188818818	1811118118	1811181888	4999999985
8881181111	1118818888	1118888188	8881111118	8188181888	1811818118	8188181181	1811818818	1811881881	8188118111	4999999985
1188818888	1188881111	8811111811	1188811118	8118181888	8118181881	1881818818	1881881181	8118118118	1881818111	4999999985
8811118888	8811181111	1188888188	8811111118	1881881888	1881818118	8118118818	8118181181	1881881881	8118118111	4999999985
1118881111	8881118888	8881188188	1118888881	1811818111	8188118118	1811881181	8188118818	8188181881	1811881888	4999999985
1118118888	1118181111	8881811811	8881888881	8188818111	1811181881	1811118818	1811181181	8188818118	8188881888	4999999985
4999999985	4999999985	4999999985	4999999985	4999999985	4999999985	4999999985	4999999985	4999999985	4999999985	4999999985
21	22	23	24	25	26	27	28	28	29	30

It is based on 2 by 2 combinations of 30 numbers:

{11111,11118,11181,11188,11811,11818,11881,11888,18111,18118,18181,18188,18811,18818,18888,  
81111,81181,81188,81811,81818,81881,81888,88111,88118,88181,88188,88811,88818,88881,88888}.

These 30 numbers are **upside-down** and **mirror looking** in pairs:  $\{88811, 11888\}$ ,  $\{11188, 88111\}$ ,  $\{11181, 18111\}$ ,  $\{81888, 88818\}$ ,  $\{18818, 81881\}$ ,  $\{18118, 81181\}$ ,  $\{11881, 18811\}$ ,  $\{88118, 81188\}$ ,  $\{11118, 81111\}$ ,  $\{18888, 88881\}$ ,  $\{11818, 81811\}$  and  $\{18188, 88181\}$ . The numbers  $\{11111, 11811, 18181, 81818, 88188, 88888\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.23 is **universal magic square**. Its magic sum is

$$S_{30 \times 30} := 149999999985 = 100000 \times 1499985 + 1499985,$$

where

$$\begin{aligned} 1499985 := & 11111 + 11118 + 11181 + 11188 + 11811 + 11818 + 11881 + 11888 + 18111 + 18118 + \\ & + 18181 + 18188 + 18811 + 18818 + 18888 + 81111 + 81181 + 81188 + 81811 + 81818 + \\ & + 81881 + 81888 + 88111 + 88118 + 88181 + 88188 + 88811 + 88818 + 88881 + 88888. \end{aligned}$$

The blocks of order 6 appearing in Example 2.23 are of equal magic sums. The blocks are constructed dividing the above 30 numbers in five parts with equal sums:

$$\begin{aligned} A &:= 11188 + 88811 + 11888 + 88111 + 11181 + 88818 = 299997 \\ B &:= 18111 + 81888 + 18818 + 81181 + 18118 + 81881 = 299997 \\ C &:= 11111 + 88888 + 11881 + 88118 + 18811 + 81188 = 299997 \\ D &:= 11811 + 88188 + 11118 + 88881 + 18888 + 81111 = 299997 \\ E &:= 11818 + 88181 + 81811 + 18188 + 18181 + 81818 = 299997 \end{aligned}$$

Based on these five groups  $A, B, C, D$  and  $E$ , the 25 magic squares of order 6 are constructed with equal magic sums  $S_{6 \times 6} := 29999999997$ . These 25 magic squares lead us to a magic square of order 30 given in Example 2.23.

## 2.14.2 Blocks of Order 5

**Example 2.24.** *The magic square of order 30 for the digits 1 and 8 is given below in three parts:*



30x30	mgc								
1888118881	1811881888	8118811818	1811181111	8118188881	1111118111	8818881118	1888881881	1111818811	8881111111
8188818118	8111881118	8188181888	8118111818	8881818888	1811811181	8888888818	1111111188	8888118818	8111111881
1181881188	8188881881	1118811188	1881881888	1811811818	1888188881	8188118111	8811818881	8888888111	1118118811
8111118111	1181881181	8188818818	8811188111	1881181888	8188111818	8111811181	1881888818	8118881118	8811811888
8888181181	1888888818	1181818118	8188818811	1188811888	1188181888	1881811818	1118818111	1881118881	8118111188
1811111111	1118118118	8888118881	1181881881	8188811881	8881188811	8818181888	1881111818	8811188818	1188181118
8111888188	8881888888	1811181881	1118181118	1181818818	8188888181	1111811118	1181181888	1188111818	1188818881
8188118888	1118811111	1888188118	8881818818	1811111188	1181818811	8188811811	8111181111	8818881888	8818111818
1881111118	1881888881	8811188888	8111881188	1888118811	8881888111	1181811881	8188888188	1888818888	1111181888
1111188811	1188181111	1881111181	1188888118	1118881181	8111811881	1888111888	1181888181	8188811111	8888188881
1888811888	8888811118	8818118888	1188118111	8881181188	8811118118	1118888181	8111888811	1181811811	8188888888
8888811818	8888188811	8811881111	1181188881	8818188818	1111881181	1188881881	8811111811	1118811118	1181888188
8118181888	8811811818	1118111118	8118818888	8818811181	1181118881	8111118118	8881118818	1188888188	8811181111
8811181118	1188811188	8881188111	1111811888	8111188811	1888811118	8888181111	1118118888	1811188881	8881811181
8818811811	1111188188	8888811111	8811888888	8118888118	8118181188	1811881181	8188118118	1881881881	1881118818
1181111188	8818888111	1111111888	8888888811	8811811118	8118881111	8118118888	1811888881	8188111181	1881818111
1188888111	8881111888	1111888811	8111111118	1888881111	8888118888	1118188881	1811111181	8881818111	1888188818
1811818188	1881181811	8118111881	8818888818	1118188111	1118818818	1811188888	1111881188	1181118118	8188188118
1881881811	8118818811	1181118111	8888111188	8111881881	1118111111	8881188118	8818181181	1811888888	8181111118
8811818818	8818111181	1888881118	1888118118	8888188188	1188888888	1188181188	8118111111	8181188811	1818888111
1188188881	8111118881	8881881181	1888811811	8811111111	1881188118	8118888188	8181111888	1818811188	1811188181
1111888818	1811181188	8111188181	1118888188	1881888888	8811811811	8181188111	1818881118	1118111881	8888818188
1118188118	1111811881	8111811811	8188111111	8888888181	8181111188	1818818881	8888118811	1111118188	8118881811
8881118811	1888188181	1811888188	1111111881	8181181118	1818888818	1888818818	8818818188	8811881811	1181181188
8881811881	8118111811	8818818811	8181118881	1818818111	8111181881	1181118188	8888881811	8818188118	1811888811
8118888181	1181118818	8181188818	1818811181	1111818118	8818118188	1111181811	1188188888	8118111888	1118818888
8818181881	8181118111	1818888881	8881181181	1188118188	8818881811	1881111111	8118888111	8111881111	1111811811
8181111181	1818818888	1188881188	1881118188	1181181811	1881888188	8811811188	1888111118	8881188181	8111881181
1818881111	8811188118	1881818188	8818181811	8188111811	8888881118	8881888811	1188811881	1888181188	1888818118
1118888888	8188118188	1188181811	1811888181	1111118881	1811111888	8811118811	8881888118	8111181181	8818881881
14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985
1	2	3	4	5	6	7	8	9	10

1188818888	1181888888	8188881181	8111888111	1181188188	1118811811	8811111888	1818818118	8181118818	1881888118
1111888888	8881188881	1181888118	1118811888	8818811111	8811188188	1188888811	8181118811	1881181188	1118188181
1888888181	8111188118	1111811181	8811188811	1111188888	1188811111	8881111118	1188181181	1811111811	8111818888
1811111881	8888111811	1888881188	1188811118	8888888118	8881188888	1111881111	8881888188	1118888881	1811818881
8118888811	8881888181	1118188188	8881181111	8811881188	1111888118	8111118888	8811111181	8188181118	8818888881
1811888111	8118111118	1888111811	1111818888	8118881181	8111181188	1888888881	1881811188	1111111181	8888811888
8818111188	8188111888	1811881111	8111188881	8118118118	1888881181	8888111181	8888818111	8811888811	8118811881
8881181118	1181188111	1881888811	1888811181	1811881881	8888118118	1118118111	8118811118	8118188181	1111181181
1181111818	1111811188	8818811888	8888118111	8188118818	1118181881	1811188818	1811811811	8888818118	8881181811
8888881888	8818811818	8111188111	1118188818	1881818811	1811118818	8881818881	8811881881	1111881811	8811118188
1118111181	8811881888	1111111818	1811118881	1881111881	8881818811	1888181118	8111181811	1188818188	1181188818
8188888118	1811118111	8118881888	8881881118	1188188181	1888111881	8111811188	8881118188	8818818881	1818881188
1181811111	8188881188	8881888818	1888111188	8818111811	8111888181	1118888111	1111181118	1818881181	8181181881
1888118111	8111888818	1118818881	8188881888	8181118188	1818811818	1181881811	1181111111	8818188188	1188111811
1188118811	8818111881	1181188181	1818881811	1181811818	8188818188	8181181888	1118888811	8111811888	1888188111
1881188818	1188118881	8818181118	1181818188	1818881888	8181181811	8188811818	8111818818	1888181881	8881818118
8111818881	1118881118	8811111188	8181111818	8188881811	1181881888	1818818188	8818888118	1181188888	8818111111
8181181111	1818888811	8111811111	1111111811	8881111188	1881881118	8811888188	8818188181	8188811118	1181818111
1818811888	1888188188	8118118188	8818888181	1188881118	8188118881	8888811811	1111881888	1188111881	8188888811
8881811811	8118818188	8188181811	1181111881	8811118881	1811888818	1111188181	1811111818	8881181888	1881118811
8811818188	1811881811	8888881881	8818118811	1118888818	8118118111	8818811881	8888181188	1118111818	1188881888
8111818881	1111118118	1881118888	1188118818	8111818111	8118811181	1181118811	1188888881	1888888118	8888111818
8818881181	1881881111	8881118111	1881181881	1888111181	8811888881	8818118818	8118188111	8811118888	8111188888
8188111118	1188811181	8888188888	1881818118	8881888881	8888818888	1188181881	1881111888	8118811188	1118881111
8811188881	1888811111	1188818818	8188181181	1811118888	1111181111	1881118118	1118188888	1881888111	8811881118
8111188188	8811181881	1811118811	1811881188	1118181111	8818811118	1881881181	1888811881	8888111111	8188111188
1118818118	1118118818	8811811881	8118188118	8888111118	1181188811	8188181188	1888118888	8111118811	1888888188
8888181881	8888818811	1188118118	8118888888	1888888811	8818111888	1811888118	8188188818	8881881111	1111818818
1111118818	1881181181	8181188881	8811811111	8111111888	1188188111	8118188888	1181818881	1811818111	1811111118
1881881188	8181118888	1818811118	8888888188	1111888111	1881111188	8118811111	8188881111	1181888818	8118111181
149999999985	149999999985	149999999985	149999999985	149999999985	149999999985	149999999985	149999999985	149999999985	149999999985
11	12	13	14	15	16	17	18	19	20



										14999999985
8888111881	8881811118	8811811181	1881188811	1188188818	8818181188	8188188181	1118181811	8111118188	8888811188	14999999985
1888181111	8118818111	1188111118	8818118881	1181181181	1881811811	1811181811	1888818188	8811888111	1818881881	14999999985
8118188818	8818181111	1181181118	8818818118	1881188188	8881881811	8888118188	8118811888	1818818818	8181111881	14999999985
1181118888	8818811188	1111181881	1188111111	1888181811	1118118188	8118188811	1818818811	8181188181	8818118118	14999999985
1111188111	8888818818	8818188888	8111881811	1811118188	1811811118	1818811881	8181111811	1181181881	1888111111	14999999985
8811818811	1181188118	1118881811	8881818188	8188181111	1818888181	8181188188	8818818818	8111888888	1188818111	14999999985
8818881188	8811181811	1888118188	1881818888	1818811811	8181111111	1111118811	1118888118	8881188818	1881188111	14999999985
1188881811	8111818188	1881188881	1818888188	8181188888	8888811881	8811181188	1111818881	1188111888	8811888818	14999999985
1118818188	1188111181	1818811111	8181188118	8811888181	1188881181	8111181118	8818188811	8118818881	8118181111	14999999985
8818118111	1818888888	8181181188	8118811811	8881118118	1888811188	1181111118	8118181118	1811818888	8188188188	14999999985
1818888118	8181181181	8118188188	1111881881	8888188111	8818881111	1811811188	8188188881	1881811111	8118818818	14999999985
8181118118	1811811111	8111118818	1118111888	1111118888	8188188111	1881811181	1881188888	8118118811	1888881811	14999999985
8188188888	1888818811	1811188811	8888888881	1881811888	1881118111	1188188118	1811811881	8888181811	1111818188	14999999985
1881188181	1881811881	8188118811	1811818818	8118181881	8118818118	8811881181	8888881188	1111188118	8818888888	14999999985
8881811188	1811181118	1118118881	8888188818	1888818111	8111111181	1111888881	8881118888	1188881111	8811111118	14999999985
1811181181	1118181188	8888188118	1888888888	8111111111	1111888188	8881111811	1188888181	8811111881	1118818811	14999999985
1188188188	1881111811	1881888181	8188111881	1811818811	8118118818	8118881881	8811818118	8888881181	1111181188	14999999985
8118888881	8888111888	8811181181	1188818811	8888811181	1188118888	1888818881	8881881881	1888111818	8111181888	14999999985
1181811181	8811818888	1888888111	1118881188	8811118818	1111188881	1881181111	8111188818	1811181118	8881811818	14999999985
8188811888	1181888881	8888881111	8111111188	8111888118	1118881881	8818818888	1881811118	1111818111	1118181181	14999999985
1881818818	8188888111	1181818888	1111111118	1111881118	1888188888	8111818118	1181181111	8188188811	8881111181	14999999985
8811181888	8188181881	8188811188	1181881111	8818888811	8881118881	8881811111	1888181181	8818111118	1811811888	14999999985
1888811818	1118881888	1811818118	8188881118	1181811118	1181111888	1188888818	1811188188	8881881188	1188188811	14999999985
1111811111	8111111818	8111881888	8118181181	8188818881	1181888811	8818188111	8811118111	1118111811	1811188118	14999999985
8111811118	8881188188	1111818118	1888181888	8118881188	8188888818	1181811888	1188111188	1118811181	8888188181	14999999985
8888818881	1888188811	1188811811	8881111818	8881881888	8811888118	8188818111	1181888111	1881181118	8111888881	14999999985
1811881118	1111188818	8881811888	8811188181	1188811818	1811181888	8888888888	8188811181	1181811188	1881818881	14999999985
8111111811	8118118881	8818818111	1811188111	1118811881	8811111818	1118181888	1111111111	8188888881	1181881118	14999999985
8881181881	1111888181	8118888818	1181111181	1118111188	8111818811	1118811818	8888181888	8818888188	8188818888	14999999985
1118188811	1188818118	8881111881	8811818111	8818188881	8888181118	1888118818	8111811818	1888881888	1181111811	14999999985
14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985	14999999985
21	22	23	24	25	26	27	28	28	29	30

This magic square is also constructed with the same number as of Example 2.23. These numbers lead us to a **universal magic square with magic sum**, i.e.,

$$S_{30 \times 30} := 149999999985 = 100000 \times 1499985 + 1499985,$$

The magic square given in Example 2.24 is constructed based on blocks of order 5. These blocks are **pandiagonal magic squares** of order 5 with different magic sums resulting in a **magic square** of order 30.

## 2.15 Magic Square of Order 31

**Example 2.25.** *The magic square of order 31 for the digits 1 and 8 is given below in three parts:*



3 x3	pan	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803
	1888118881	8181811188	1181111888	8818811118	1111118888	8888811181	8811888818	8118811818	8118118188	1811811881
158181881803	1118881818	8111881118	1888188111	8181888811	1181181111	8818888881	1111118111	8888881888	8811881811	8118888181
158181881803	1188811811	8811118881	1118811188	8111811888	1888111118	8181818888	1181111181	8818888818	1111111818	8888818188
158181881803	1111888188	8881181818	1188881118	8811188111	1118888811	8111881111	1888188881	8181818111	1181181888	8818881811
158181881803	1888811111	8111111811	1111818881	8881111188	1188811888	8811111118	1118818888	8111811181	1888188818	8181811818
158181881803	1118188888	8888188188	1888881818	8111181118	1111888111	8881188811	1188881111	8811188881	1118818111	8111881888
158181881803	8881888118	1811111111	1118111811	8888118881	1888811188	8111111888	1111811118	8881118888	1188811181	8811188818
158181881803	1181881188	8188888888	8881888188	1811181818	1118181118	8888188111	1888888811	8111181111	1111888881	8881118111
158181881803	1818881181	8181188118	1181811111	8188811811	8881818881	1811111188	1118111888	8888111118	1888818888	8111111181
158181881803	1188118118	8818181188	1818888888	8181188188	1181881818	8188881118	8881888111	1811188811	1118181111	8888188881
158181881803	1881881881	1881181181	1188188118	8818111111	1818811811	8181118881	1181811188	8188811888	8881811118	1811118888
158181881803	1811818818	8188118118	1881881188	1881188888	1188188188	8818181818	1818881118	8181188111	1181888811	8188881111
158181881803	8118818811	8118181881	1811881181	8188188118	1881811111	1881111811	1188118881	8818111188	1818811888	8181111118
158181881803	8888811881	8811818818	8118818118	8118181188	1811888888	8188188188	1881881818	1881181118	1188188111	8818188811
158181881803	8818888181	1111118811	8888881881	8811881181	8118888118	8118111111	1811811811	8188118881	1881811188	1881111888
158181881803	8181818188	1181111881	8818818818	1111118118	8888881188	8811888888	8118888188	8118181818	1811881118	8188188111
158181881803	8111881811	1888188181	8181818811	1181181881	8818881181	1111188118	8888811111	8811811811	8118818881	8118111188
158181881803	8811111818	1118818188	8111811881	1888118818	8181818118	1181181188	8818888888	1111188188	8888881818	8811881118
158181881803	8881181888	1188881811	8811188181	1118818811	8111881881	1888181181	8181888118	1181111111	8818811811	1111118881
158181881803	8111188818	1111811818	8881118188	1188811881	8811118818	1118818118	8111881188	1888188888	8181888188	1181181818
158181881803	8888118111	1888881888	8111181811	1111888181	8881118811	1188881881	8811181181	1118888118	8111811111	1888111811
158181881803	1811111181	1118188818	8888111818	1888818188	8111111881	1111818818	8881118118	1188881188	8811188888	1118888188
158181881803	8188888881	8881818111	1811181888	1118181811	8888188181	1888818811	8111181881	1111881181	8881188118	1188811111
158181881803	8181118888	1181811181	8188888818	8881811818	1811118188	1118111881	8888118818	1888818118	8111181188	1111888888
158181881803	8818181111	1818888881	8181118111	1181881888	8188881811	8881888181	1811118811	1118181881	8888181181	1888888118
158181881803	1881111118	1188118888	8818111181	1818888818	8181111818	1181818188	8188811881	8881818818	1811118118	1118181188
158181881803	8188188811	1881881111	1881188881	1188118111	8818181888	1818881811	8181188181	1181818811	8188881881	8881881181
158181881803	8118111888	1811811118	8188118888	1881811181	1881188818	1188111818	8818118188	1818811881	8181118818	1181818118
158181881803	8811888111	8118888811	8118181111	1811888881	8188118111	1881881888	1881181811	1188188181	8818118811	1818881881
158181881803	1111111188	8888811888	8811811118	8118818888	8118111181	1811888818	8188111818	1881818188	1881111881	1188118818
158181881803	1181181118	8818888111	1111188811	8888881111	8811888881	8118818111	8118181888	1811881811	8188188181	1881818811
	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803
	1	2	3	4	5	6	7	8	9	10



158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803
8188118818	1881818118	1881181188	1188188888	8818188188	1818881818	8181181118	1181888111	8188888811	8881881111	1811188881
8118118811	1811881881	8188181181	1881888118	1881111111	1188111811	8818118881	1818811188	8181111888	1181811118	8188818888
8811811881	8118818818	8118118118	1811881188	8188188888	1881888188	1881181818	1188181118	8818188111	1818888811	8181181111
1111188181	8888818811	8811881881	8118881181	8118188118	1811811111	8188111811	1881818881	1881111188	1188111888	8818111118
1181118188	8818811881	1111118818	8888818118	8811881188	8118888888	8118188188	1811881818	8188181118	1881888111	1881188811
1888181811	8181888181	1181118811	8818881881	1111181181	8888888118	8811811111	8118811811	8118118881	1811811188	8188111888
1118811818	8111818188	1888111881	8181818818	1181118118	8818881188	1111188888	8888888188	8811881818	8118881118	8118188111
1188881888	8811181811	1118888181	8111818811	1888181881	8181881181	1181188118	8818811111	1111111811	8888818881	8811811188
1111888818	8881111818	1188818188	8811111881	1118818818	8111818118	1888181188	8181888888	1181188188	8818881818	1111181118
1888818111	8111181888	1111881811	8881188181	1188818811	8811181881	1118881181	8111888118	1888111111	8181811811	1181118881
1118111181	8888188818	1888811818	8111118188	1111811881	8881118818	1188818118	8811181188	1118888888	8111888188	1888181818
8881888881	1811118111	1118181888	8888181811	1888888181	8111118811	1111881881	8881181181	1188888118	8811111111	1118811811
1181818888	8188811181	8881888818	1811111818	1118118188	8888111881	1888818818	8111118118	1111881188	8881188888	1188888188
1818881111	8181188881	1181818111	8188881888	8881881811	1811188181	1118118811	8888181881	1888881181	8111188118	1111811111
1188111118	8818118888	1818811181	8181188818	1181811818	8188818188	8881811881	1811118818	1118118118	8888181188	1888888888
1881888811	1881181111	1188188881	8818118111	1818881888	8181181811	1181888181	8188818811	8881881881	1811181181	1118188118
1811811888	8188111118	1881818888	1881111181	1188188818	8818111818	1818818188	8181111881	1181818818	8188818118	8881881188
8118888111	8118188811	1811881111	8188188881	1881818111	1881181888	1188181811	8818188181	1818818811	8181181881	1181881181
8888811188	8811811888	8118811118	8118118888	1811811181	8188188818	1881811818	1881118188	1188111881	8818118818	1818818118
8818881118	1111188111	8888888811	8811881111	8118888881	8118118111	1811881888	8188181811	1881888181	1881118811	1188181881
8181818881	1181111188	8818811888	1111111118	8888818888	8811811181	8118888818	8118111818	1811818188	8188111881	1881818818
8111881818	1888181118	8181888111	1181188811	8818881111	1111188881	8888818111	8811881888	8118881811	8118188181	1811818811
8811111811	1118818881	8111811188	1888111888	8181811118	1181118888	8818811181	1111188818	8888811818	8811818188	8118811881
8881188188	1188881818	8811181118	1118888111	8111888811	1888181111	8181888881	1181118111	8818881888	1111181811	8888888181
8111111111	1111811811	8881118881	1188811188	8811111888	1118811118	8111818888	1888111181	8181888818	1181111818	8818818188
8888188888	1888888188	8111181818	1111881118	8881188111	1188888811	8811181111	1118888881	8111818111	1888181888	8181881811
1811188118	1118111111	8888111811	1888818881	8111111188	1111811888	8881111118	1188818888	8811111181	1118888818	8111811818
8188881188	8881888888	1811188188	1118181818	8888181118	1888888111	8111188811	1111881111	8881188881	1188818111	8811181888
8181181181	1181888118	8188811111	8881811811	1811118881	1118111188	8888111888	1888811118	8111118888	1111811181	8881188818
8818118118	1818881188	8181188888	1181888188	8188881818	8881881118	1811188111	1118188811	8888181111	1888888881	8111118111
1881181881	1188181181	8818188118	1818811111	8181111811	1181818881	8188811188	8881811888	1811111118	1118118888	8888111181
158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803
11	12	13	14	15	16	17	18	19	20	21



158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803
1118118111	8888181888	1888881811	8111188181	1111818811	8881181881	1188881181	8811188118	1118811111	8111811811	158181881803
8881811181	1811188818	1118111818	8888118188	1888811881	8111118818	1111818118	8881181188	1188888888	8811188188	158181881803
1181888881	8188818111	8881881888	1811181811	1118188181	8888118811	1888881881	8111181181	1111888118	8881111111	158181881803
1818818888	8181111181	1181888818	8188811818	8881818188	1811111881	1118118818	8888118118	1888881188	8111188888	158181881803
1188181111	8818188881	1818818111	8181181888	1181881811	8188888181	8881818811	1811181881	1118181181	8888188118	158181881803
1881811118	1881118888	1188111181	8818188818	1818811818	8181118188	1181811881	8188818818	8881818118	1811181188	158181881803
1811888811	8188181111	1881888881	1881118111	1188181888	8818181811	1818888181	8181118811	1181881881	8188881181	158181881803
8118811888	8118111118	1811818888	8188111181	1881888818	1881111818	1188118188	8818111881	1818818818	8181118118	158181881803
8888888111	8811888811	8118881111	8118188881	1811818111	8188181888	1881881811	1881188181	1188118811	8818181881	158181881803
8818811188	1111111888	8888811118	8811818888	8118811181	8118188818	1811811818	8188118188	1881811881	1881118818	158181881803
8181881118	1181188111	8818888811	1111181111	8888888881	8811818111	8118881888	8118181811	1811888181	8188118811	158181881803
8111818881	1888111188	8181811888	1181111118	8818818888	1111111181	8888888818	8811811818	8118818188	8118111881	158181881803
8811181818	1118881118	8111888111	1888188811	8181881111	1181188881	8818818111	1111181888	8888881811	8811888181	158181881803
8881111811	1188818881	8811111188	1118811888	8111811118	1888118888	8181811181	1181188818	8818811818	1111118188	158181881803
8111188188	1111881818	8881181118	1188888111	8811188811	1118881111	8111888881	1888118111	8181881888	1181181811	158181881803
8888111111	1888811811	8111118881	1111811188	8881111888	1188811118	8811118888	1118811181	8111888818	1888111818	158181881803
1811188888	1118188188	8888181818	1888881118	8111188111	1111888811	8881181111	1188888881	8811118111	1118881888	158181881803
8188888118	8881811111	1811111811	1118118881	8888111188	1888811888	8111111118	1111818888	8881111181	1188888818	158181881803
8181181188	1181888888	8188888188	8881881818	1811181118	1118188111	8888188811	1888881111	8111188881	1111818111	158181881803
8818181181	1818888118	8181111111	1181811811	8188818881	8881811188	1811111888	1118111118	8888118888	1888811181	158181881803
1881118118	1188181188	8818188888	1818888188	8181181818	1181881118	8188888111	8881888811	1811181111	1118188881	158181881803
8188181881	1881881181	1881188118	1188111111	8818111811	1818818881	8181111188	1181811888	8188811118	8881818888	158181881803
8118118818	1811818118	8188181188	1881888888	1881188188	1188181818	8818181118	1818888111	8181188811	1181881111	158181881803
8811818811	8118881881	8118181181	1811888118	8188111111	1881811811	1881118881	1188111188	8818111888	1818811118	158181881803
1111111881	8888818818	8811818118	8118881188	8118188888	1811888188	8188181818	1881881118	1881188111	1188188811	158181881803
1181188181	8818818811	1111181881	8888881181	8811888118	8118811111	8118111811	1811818881	8188111188	1881811888	158181881803
1888118188	8181811881	1181118818	8818818118	1111181188	8888888888	8811888188	8118881818	8118181118	1811888111	158181881803
1118881811	8111888181	1888118811	8181881881	1181181181	8818888118	1111111111	8888811811	8811818881	8118811188	158181881803
1188811818	8811118188	1118811881	8111818818	1888118118	8181881188	1181188888	8818888188	1111181818	8888881118	158181881803
1111881888	8881181811	1188888181	8811118811	1118881881	8111881181	1888188118	8181811111	1181111811	8818818881	158181881803
1888888818	8111111818	1111818188	8881111881	1188818818	8811118118	1118881188	8111888888	1888188188	8181881818	158181881803
158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803	158181881803
22	23	24	25	26	27	28	29	30	31	

It is based on 2 by 2 combinations of 31 numbers:

$\{11111, 11118, 11181, 11188, 11811, 11818, 11881, 11888, 18111, 18118, 18188, 18811, 18818, 18881, 18888, 81111, 81118, 81181, 81188, 81811, 81818, 81881, 81888, 88111, 88118, 88181, 88188, 88811, 88818, 88881, 88888\}$ .

These 31 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{11181, 18111\}$ ,  $\{11188, 88111\}$ ,  $\{11818, 81811\}$ ,  $\{11881, 18811\}$ ,  $\{11888, 88811\}$ ,  $\{18118, 81181\}$ ,  $\{18188, 88181\}$ ,  $\{18818, 81881\}$ ,  $\{81188, 88118\}$ ,  $\{18888, 88881\}$  and  $\{81888, 88818\}$ . The numbers  $\{11111, 11811, 18881, 81118, 81818, 88188, 88888\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.25 is **universal pandiagonal magic square**. Its **magic sum** is

$$S_{31 \times 31} := 158181881803 = 100000 \times 1581803 + 1581803,$$

where

$$\begin{aligned} 1581803 := & 11111 + 11118 + 11181 + 11188 + 11811 + 11818 + 11881 + 11888 + 18111 + 18118 + 18188 + \\ & + 18811 + 18818 + 18881 + 18888 + 81111 + 81118 + 81181 + 81188 + 81811 + 81818 + \\ & + 81881 + 81888 + 88111 + 88118 + 88181 + 88188 + 88811 + 88818 + 88881 + 88888. \end{aligned}$$

## 2.16 Magic Square of Order 32

### 2.16.1 Blocks of Order 4

**Example 2.26.** *The magic square of order 32 for the digits 1 and 8 is given below in three parts:*



32x32	pan	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984
	1888181118	8111888888	1111111111	8888818881	1818181118	8181888888	1181111111	8818818881	1888181111	8111888881
15999999984	1111118881	8888811111	1888188888	8111881118	1181118881	8818811111	1818188888	8181881118	1111118888	8888811118
15999999984	8888888888	1111181118	8111818881	1888111111	8818888888	1181181118	8181818881	1818111111	8888888881	1111181111
15999999984	8111811111	1888118881	8888881118	1111188888	8181811111	1818118881	8818881118	1181188888	8111811118	1888118888
15999999984	1888181818	8111888188	1111111811	8888818181	1818181818	8181888188	1181111811	8818818181	1888188111	8111888811
15999999984	1111118181	8888811811	1888188188	8111881818	1181118181	8818811811	1818188188	8181881818	1111118888	8888811188
15999999984	8888888188	1111181818	8111818181	1888111811	8818888188	1181181818	8181818181	1818111811	8888888811	1111188111
15999999984	8111811811	1888118181	8888881818	1111188188	8181811811	1818118181	8818881818	1181188188	8111811188	1888111888
15999999984	1888881118	8111188888	1111811111	8888118881	1188881118	8811188888	1118811111	8881118881	1888881111	8111188881
15999999984	1111818881	8888111111	1888888888	8111181118	1118818881	8881111111	1188888888	8811181118	1111818888	8888111118
15999999984	8888188888	1111881118	8111118881	1888811111	8881188888	1118881118	8811118881	1188811111	8888188881	1111881111
15999999984	8111111111	1888818881	8888181118	1111888888	8811111111	1188818881	8881181118	1118888888	8111111118	1888818888
15999999984	1888881818	8111188188	1111811811	8888118181	1188881818	8811188188	1118811811	8881118181	1888888111	8111188811
15999999984	1111818181	8888111811	1888888188	8111181818	1118818181	8881111811	1188888188	8811181818	1111811888	8888111188
15999999984	8888188188	1111881818	8111118181	1888811811	8881188188	1118881818	8811118181	1188811811	8888188811	1111888111
15999999984	8111111811	1888818181	8888181818	1111888188	8811111811	1188818181	8881181818	1118888188	8111111188	1888811888
15999999984	1818881118	8181188888	1118111111	8881818881	1811181118	8188888888	1181811111	8818118881	1818881111	8181188881
15999999984	1118118881	8881811111	1818888888	8181181118	1181818881	8818111111	1811188888	8188881118	1118118888	8881811118
15999999984	8881888888	1118181118	8181118881	1818811111	8818188888	1181881118	8188818881	1811111111	8881888881	1118181111
15999999984	8181111111	1818818881	8881881118	1118188888	8188811111	1811118881	8818181118	1181888888	8181111118	1818818888
15999999984	1818881818	8181188188	1118111811	8881818181	1811181818	8188888188	1181811811	8818118181	1818888111	8181188811
15999999984	1118118181	8881811811	1818888188	8181181818	1181818181	8818111811	1811188188	8188881818	1118111888	8881811188
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15999999984	8181111811	1818818181	8881881818	1118188188	8188811811	1811118181	8818181818	1181888188	8181111188	1818811888
15999999984	1881881118	8118188888	1188111111	8811818881	1881181118	8118888888	1811811111	8188118881	1881881111	8118188881
15999999984	1188118881	8811811111	1881888888	8118181118	1811818881	8188111111	1881188888	8118881118	1188118888	8811811118
15999999984	8811888888	1188181118	8118118881	1881811111	8188188888	1811881118	8118818881	1881111111	8811888881	1188181111
15999999984	8118111111	1881818881	8811881118	1188188888	8118811111	1881118881	8188181118	1811888888	8118111118	1881818888
15999999984	1881881818	8118188188	1188111811	8811818181	1881181818	8118888188	1811811811	8188118181	1881888111	8118188811
15999999984	1188118181	8811811811	1881888188	8118181818	1811818181	8188111811	1881188188	8118881818	1188111888	8811811188
15999999984	8811888188	1188181818	8118118181	1881811811	8188188188	1811881818	8118818181	1881111811	8811888811	1188188111
15999999984	8118111811	1881818181	8811881818	1188188188	8118811811	1881118181	8188181818	1811888188	8118111188	1881811888
	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984
	1	2	3	4	5	6	7	8	9	10



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111111118	8888818888	1818181111	8181888881	1181111118	8818818888	1888181811	8111888818	1111111181	8888818188	1818181811	8181888818
1888188881	8111881111	1181118888	8818811118	1818188881	8181881111	1111118188	8888811181	1888188818	8111881811	1181118188	8818811181
8111818888	1888111118	8818888881	1181181111	8181818888	1818111118	88888888818	1111181811	8111818188	1888111181	8818888818	1181181811
8888881111	1111188881	8181811118	1818118888	8818881111	1181188881	8111811181	1888118188	8888881811	1111188818	8181811181	1818118188
1111111188	8888811888	1818188111	8181888811	1181111188	8818811888	1888181888	8111888181	1111111818	8888818111	1818181888	8181888181
1888188811	8111888111	1181111888	8818811188	1818188811	8181888111	1111118111	8888811818	1888188181	8111881888	1181118111	8818811818
8111811888	1888111188	8818888811	1181188111	8181811888	1818111188	8888888181	1111181888	8111818111	1888111818	8818888181	1181181888
8888888111	1111188811	8181811188	1818111888	8818888111	1181188811	8111811818	1888118111	8888881888	1111188181	8181811818	1818118111
1111811118	8888118888	1188881111	8811188881	1118811118	8881118888	1888881811	8111188818	1111811181	8888118188	1188881811	8811188818
1888888881	8111181111	1118818888	8881111118	1188888881	8811181111	1111818188	8888111181	1888888818	8111181811	1118818188	8881111181
8111118888	1888811118	8881188881	1118881111	8811118888	1188811118	8888188818	1111881811	8111118188	1888811181	8881188818	1118881811
8888181111	1111888881	8811111118	1188818888	8881181111	1118888881	8111111181	1888818188	8888181811	1111888818	8811111181	1188818188
1111811188	8888111888	1188888111	8811188811	1118811188	8881111888	1888881888	8111188181	1111811818	8888118111	1188881888	8811188181
1888888811	8111188111	1118811888	8881111188	1188888811	8811188111	1111818111	8888111818	1888888181	8111181888	1118818111	8881111818
8111111888	1888811188	8881188811	1118888111	8811111888	1188811188	8888188181	1111881888	8111118111	1888811818	8881188181	1118881888
8888188111	1111888811	8811111188	1188811888	8881188111	1118888811	8111111818	1888818111	8888181888	1111888181	8811111818	1188818111
1118111118	8881818888	1811181111	8188888881	1181811118	8818118888	1818881811	8181188818	1118111181	8881818188	1811181811	8188888818
1818888881	8181181111	1181818888	8818111118	1811188881	8188881111	1118118188	8881811181	1818888818	8181181811	1181818188	8818111181
8181118888	1818811118	8818188881	1181881111	8188818888	1811111118	8881888818	1118181811	8181118188	1818811181	8818188818	1181881811
8881881111	1118188881	8188811118	1811118888	8818181111	1181888881	8181111181	1818818188	8881881811	1118188818	8188811181	1811118188
1118111188	8881811888	1811188111	8188888811	1181811188	8818111888	1818881888	8181188181	1118111818	8881818111	1811181888	8188888181
1818888811	8181188111	1181811888	8818111188	1811188811	8188888111	1118118111	8881811818	1818888181	8181181888	1181818111	8818111818
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8881888111	1118188811	8188811188	1811111888	8818188111	1181888811	8181111818	1818818111	8881881888	1118188181	8188811818	1811118111
1188111118	8811818888	1881181111	8118888881	1811811118	8188118888	1881881811	8118188818	1188111181	8811818188	1881181811	8118888818
1881888881	8118181111	1811818888	8188111118	1881188881	8118881111	1188118188	8811811181	1881888818	8118181811	1811818188	8188111181
8118118888	1881811118	8188188881	1811881111	8118818888	1881111118	8811888818	1188181811	8118118188	1881811181	8188188818	1811881811
8811881111	1188188881	8118811118	1881118888	8188181111	1811888881	8118111181	1881818188	8811881811	1188188818	8118811181	1881118188
1188111188	8811811888	1881188111	8118888811	1811811188	8188111888	1881881888	8118188181	1188111818	8811818111	1881181888	8118888181
1881888811	8118188111	1811811888	8188111188	1881188811	8118888111	1188118111	8811811818	1881888181	8118181888	1811818111	8188111818
8118111888	1881811188	8188188811	1811888111	8118811888	1881111188	8811888181	1188181888	8118118111	1881811818	8188188818	1811881888
8811888111	1188188811	8118811188	1881111888	8188188111	1811888811	8118111818	1881818111	8811881888	1188188181	8118811818	1881118111
15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984	15999999984
11	12	13	14	15	16	17	18	19	20	21	22



159999999984	159999999984	159999999984	159999999984	159999999984	159999999984	159999999984	159999999984	159999999984	159999999984	159999999984
118111181	8818818188	1888181181	8111888118	1111111881	8888818818	1818181181	8181888118	1181111881	8818818818	159999999984
1818188818	8181881811	1111118818	8888811881	1888188118	8111881181	1181118818	8818811881	1818188118	8181881181	159999999984
8181818188	1818111181	8888888118	1111181181	8111818818	1888111881	8818888118	1181181181	8181818818	1818111881	159999999984
8818881811	1181188818	8111811881	1888118818	8888881181	1111188118	8181811881	1818118818	8818881181	1181188118	159999999984
1181111818	8818818111	1888181188	8111881881	1111118118	8888818811	1818181188	8181881881	1181118118	8818818811	159999999984
1818188181	8181881888	1111118811	8888818118	1888181881	8111881188	1181118811	8818818118	1818181881	8181881188	159999999984
8181818111	1818111818	8888881881	1111181188	8111818811	1888118118	8818881881	1181181188	8181818811	1818118118	159999999984
8818881888	1181188181	8111818118	1888118811	8888881188	1111181881	8181818118	1818118811	8818881188	1181181881	159999999984
1118811181	8881118188	1888881181	8111188118	1111811881	8888118818	1188881181	8811188118	1118811881	8881118818	159999999984
1188888818	8811181811	1111818818	8888111881	1888888118	8111181181	1118818818	8881111881	1188888118	8811181181	159999999984
8811118188	1188811181	8888188118	1111881181	8111118818	1888811881	8881188118	1118881181	8811118818	1188811881	159999999984
8881181811	1118888818	8111111881	1888818818	8888181181	1111888118	8811111881	1188818818	8881181181	1118888118	159999999984
1118811818	8881118111	1888881188	8111181881	1111818118	8888118811	1188881188	8811181881	1118818118	8881118811	159999999984
1188888181	8811181888	1111818811	8888118118	1888881881	8111181188	1118818811	8881118118	1188881881	8811181188	159999999984
8811118111	1188811818	8888181881	1111881188	8111118811	1888818118	8881181881	1118881188	8811118811	1188818118	159999999984
8881181888	1118888181	8111118118	1888818811	8888181188	1111881881	8811118118	1188818811	8881181188	1118881881	159999999984
1181811181	8818118188	1818881181	8181188118	1118111881	8881818818	1811181181	8188888118	1181811881	8818118818	159999999984
1811188818	8188881811	1118118818	8881811881	1818888118	8181181181	1181818818	8818111881	1811188118	8188881181	159999999984
8188818188	1811111181	8881888118	1118181181	8181118818	1818811881	8818188118	1181881181	8188818818	1811111881	159999999984
8818181811	1181888818	8181111881	1818818818	8881881181	1118188118	8188811881	1811118818	8818181181	1181888118	159999999984
1181811818	8818118111	1818881188	8181181881	1118118118	8881818811	1811181188	8188881881	1181818118	8818118811	159999999984
1811188181	8188881888	1118118811	8881818118	1818881881	8181181188	1181818811	8818118118	1811181881	8188881188	159999999984
8188818111	1811111818	8881881881	1118181188	8181118811	1818818118	8818181881	1181881188	8188818811	1811118118	159999999984
8818181888	1181888181	8181118118	1818818811	8881881188	1118181881	8188818118	1811118811	8818181188	1181881881	159999999984
1811811181	8188118188	1881881181	8118188118	1188111881	8811818818	1881181181	8118888118	1811811881	8188118818	159999999984
1881188818	8118881811	1188118818	8811811881	1881888118	8118181181	1811818818	8188111881	1881188118	8118881181	159999999984
8118818188	1881111181	8811888118	1188181181	8118118818	1881811881	8188188118	1811881181	8118818818	1881111881	159999999984
8188181811	1811888818	8118111881	1881818818	8811881181	1188188118	8118811881	1881118818	8188181181	1811888118	159999999984
1811811818	8188118111	1881881188	8118181881	1188118118	8811818811	1881181188	8118881881	1811818118	8188118811	159999999984
1881188181	8118881888	1188118811	8811818118	1881881881	8118181188	1811818811	8188118118	1881181881	8118881188	159999999984
8118818111	1881111818	8811881881	1188181188	8118118811	1881818118	8188181881	1811881188	8118818811	1881118118	159999999984
8188181888	1811888181	8118118118	1881818811	8811881188	1188181881	8118818118	1881118811	8188181188	1811881881	159999999984
159999999984	159999999984	159999999984	159999999984	159999999984	159999999984	159999999984	159999999984	159999999984	159999999984	159999999984
23	24	25	26	27	28	29	30	31	32	

It is based on 2 by 2 combinations of 32 numbers:

$\{11111, 11118, 11181, 11188, 11811, 11818, 11881, 11888, 18111, 18118, 18181, 18188, 18811, 18818, 18881, 18888, 81111, 81118, 81181, 81188, 81811, 81818, 81881, 81888, 88111, 88118, 88181, 88188, 88811, 88818, 88881, 88888\}$ .

These 32 numbers are **upside-down** and **mirror looking** in pairs:  $\{11118, 81111\}$ ,  $\{18888, 88881\}$ ,  $\{11188, 88111\}$ ,  $\{11888, 88811\}$ ,  $\{11881, 18811\}$ ,  $\{81188, 88118\}$ ,  $\{11181, 18111\}$ ,  $\{81888, 88818\}$ ,  $\{11818, 81811\}$ ,  $\{18188, 88181\}$ ,  $\{18118, 81181\}$  and  $\{18818, 81881\}$ . The numbers  $\{11111, 11811, 88188, 88888, 18181, 18881, 81118, 81818\}$  are already **upside-down** and **mirror looking** as they are **palindromes**. Thus the magic square given in Example 2.26 is **universal magic square**. Its **magic sum** is

$$S_{32 \times 32} := 159999999984 = 100000 \times 1599984 + 1599984,$$

where

$$\begin{aligned} 1599984 := & 11111 + 11118 + 11181 + 11188 + 11811 + 11818 + 11881 + 11888 + \\ & + 18111 + 18118 + 18181 + 18188 + 18811 + 18818 + 18881 + 18888 + \\ & + 81111 + 81118 + 81181 + 81188 + 81811 + 81818 + 81881 + 81888 + \\ & + 88111 + 88118 + 88181 + 88188 + 88811 + 88818 + 88881 + 88888. \end{aligned}$$

The blocks of order 4 appearing in Example 2.26 are of equal magic sums. The blocks are constructed dividing the above 32 numbers in eight parts with equal sums:

$$A := 11118 + 18888 + 81111 + 88881 = 199998$$

$$E := 18181 + 18881 + 81118 + 81818 = 199998$$

$$B := 11188 + 11888 + 88111 + 88811 = 199998$$

$$F := 11818 + 18188 + 81811 + 88181 = 199998$$

$$C := 11881 + 18811 + 81188 + 88118 = 199998$$

$$G := 18118 + 18818 + 81181 + 81881 = 199998$$

$$D := 11181 + 18111 + 81888 + 88818 = 199998$$

$$H := 11111 + 11811 + 88188 + 88888 = 199998$$

Based on these eight groups  $A, B, C, D, E, F, G$  and  $H$ , gives us 64 magic squares of order 4 with equal **magic sums**  $S_{4 \times 4} := 19999999998$ . These 64 magic squares of order 4 lead us to a **pandiagonal magic square** of order 32 given in Example 2.26.

## 2.16.2 Bimagic Square

**Example 2.27.** *The magic square of order 32 for the digits 1 and 8 is given below in three parts:*



32x32-bi	pan	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984
	111818881888	888118118111	888111118188	111811881811	111888118181	888188881818	888181881881	111881118118	118118881188	881818118811
1599999999984	111888118118	888188881881	888181881818	111881118181	111818881811	888118118188	888111118111	111811881888	118188118818	881888881181
1599999999984	111811118111	888111881888	888118881811	111818118188	111881881818	888181118181	888188118118	111888881881	118111118811	881811881188
1599999999984	111881881881	888181118118	888188118181	111888881818	111811118188	888111881811	888118881888	111818118111	118181881181	881881118818
1599999999984	888111881818	111811118181	111818118118	888118881881	888181118111	111881881888	111888881811	888188118188	881811881118	118111118881
1599999999984	888181118188	111881881811	111888881888	888188118111	888111881881	111811118118	111818118181	888118881818	881881118888	118181881111
1599999999984	888118118181	111818881818	111811881881	888111118118	888188881888	111888118111	111881118188	888181881811	881818118881	118118881118
1599999999984	888188881811	111888118188	111881118111	888181881888	888118118118	111818881881	111811881818	888111118181	881888881111	118188118888
1599999999984	111118888188	888818111811	888811111888	111111888111	111188111881	888888888118	888881888181	111181111818	118818888888	881118111111
1599999999984	111188111818	888888888181	888881888118	111181111881	111118888111	888818111888	888811111811	111111888188	118888111118	881188888881
1599999999984	111111111811	888811888188	888818888111	111118111888	111181888118	888881111881	888888111818	111188888181	118811111111	881111888888
1599999999984	111181888181	888881111818	888888111881	111188888118	111111111888	888811888111	888818888188	111118111811	118881888881	881181111118
1599999999984	888811888118	111111111881	111118111818	888818888181	888881111811	111181888188	111188888111	888888111888	881111888818	118811111181
1599999999984	888881111888	111181888111	111188888188	888888111811	888811888181	111111111818	111118111881	888818888118	881181111188	118881888811
1599999999984	888818111881	111118888118	111111888181	888811111818	888888888188	111188111811	111181111888	888881888111	881118111181	118818888818
1599999999984	888888888111	111188111888	111181111811	888881888188	888818111818	111118888181	111111888118	888811111881	881188888811	118888111188
1599999999984	118818881188	881118118811	881111118888	118811881111	118888118881	881188881118	881181881181	118881118818	111118881888	888818118111
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1599999999984	118881881181	881181118818	881188118881	118888881118	118811118888	881111881111	881118881188	118818118811	111181881881	888881118118
1599999999984	881111881118	118811118881	118818118818	881118881181	881181118811	118881881188	118888881111	881188118888	888811881818	111111118181
1599999999984	881181118888	118881881111	118888881188	881188118811	881111881181	118811118818	118818118881	881118881118	888881118188	111181881811
1599999999984	881118118881	118818881118	118811881181	881111118818	881188881188	118888118811	118881118888	881181881111	888818118181	111118881818
1599999999984	881188881111	118888118888	118881118811	881181881188	881118118818	118818881181	118811881118	881111118881	888888881811	111188118188
1599999999984	118118888888	881818111111	881811111188	118111888811	118188111181	881888888818	881881888881	118181111118	111818888188	888118111811
1599999999984	118188111118	881888888881	881881888818	118181111181	118118888811	881818111188	881811111111	118111888888	111888111818	888188888181
1599999999984	118111111111	881811888888	881818888811	118118111188	118181888818	881881111181	881888111118	118188888881	111811111811	888111888188
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1599999999984	881811888818	118111111181	118118111118	881818888881	881881111111	118181888888	118188888811	881888111188	888111888118	111811111881
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1599999999984	881818111181	118118888818	118111888881	881811111118	881888888888	118188111111	118181111188	881881888811	888118111881	111818888118
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	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984
	1	2	3	4	5	6	7	8	9	10



1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984
88181118888	118111881111	118188118881	881888881118	881881881181	118181118818	111118888888	888818111111	888811111188	11111888811	111188111181	888888888818
881881881118	118181118881	118118881111	881818118888	881811118811	118111881188	111188111118	888888888881	888881888818	111181111181	111118888811	888818111188
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881888118881	118188881118	118111118888	881811881111	881818881188	118118118811	111181888881	888881111118	888888111181	111188888818	111111111188	888811888811
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881181888818	118881111181	118818888811	881118111188	881111111111	118811888888	111888118818	888188881181	888181881118	111881118881	111818881111	888118118888
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118818111118	881118888881	881181111111	118881888888	118888888811	881188111188	888111881118	111811118881	111818118818	888118881181	888181118811	111881881188
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118811888881	881111111118	881188888888	118888111111	118881111188	881181888811	888118118881	111818881118	111811881181	888111118818	888188881188	111888118811
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888811118188	111111881811	111188118181	888888881818	888881881881	111181118118	118118888188	881818111811	88181111888	118111888111	118188111881	881888888118
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111118118118	888818881881	888881118111	111181881888	111188881811	888888118188	881811888118	118111111881	118118111818	881818888181	881881111811	118181888188
111188881888	888888118111	888811881881	111111118118	111118118181	888818881818	881881111888	118181888111	118188888188	881888111811	881811888181	118111111818
111111881881	888811118118	888888888188	111188118111	111181118188	888881881811	881818111881	118118888118	118111888181	881811111818	881888888188	118188111811
111181118111	888881881888	888818118118	111118881881	111111881818	888811118181	881888888111	118188111888	118181111811	881881888188	881818111818	118118888181
888111111888	111811888111	111888111881	888188888118	888181888181	111881111818	118818881888	881118118111	881111118188	118811881811	118888118181	881188881818
888181888118	111881111881	111818888111	888118111888	888111111811	111811888188	118888118118	881188881881	881181888188	118881118181	118818881811	881118118188
888118888111	111818111888	111881888118	888181111881	888188111818	111888888181	118811118111	881111881888	881118881811	118818118188	118881881818	881181118181
888188111881	111888888118	111811111888	888111888111	888118888188	111818111811	118881881881	881181118118	881188118181	118888881818	118811118188	881111881811
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111881111811	888181888188	888118111818	111818888181	111811888118	888111111881	881188881811	118888118188	118881118111	881181881888	881118118118	118818881881
1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984	1599999999984
11	12	13	14	15	16	17	18	19	20	21	22



599999999984	599999999984	599999999984	599999999984	599999999984	599999999984	599999999984	599999999984	599999999984	599999999984	599999999984
88888 88888	1118 11118	188 8888 88	88 118 118 1	88 11111 888	188 1888 11	18888 1188	88 188888 18	88 18 888 8	1888 1118 8	599999999984
8888 111111	1111 888888	18888 118 8	88 188888 8	88 18 888 18	1888 11188	188 8888 11	88 118 11888	88 11111 8 1	188 1888 88	599999999984
888888 11118	111 8888888	188 11118 1	88 111 888 88	88 11 8888 11	188 8 11888	1888 888 18	88 18 1188	88 188 118 8	18888888 8	599999999984
8888 8888888	11118 11111	1888 888 8	88 18 1118 8	88 188 1188	18888888 18	188 1111888	88 111 888 11	88 11 8888 88	188 8 118 1	599999999984
111888888 1	888888 11188	88 111 888 18	188 111188	188 8 118 8	88 11 8888 8	88 18 1118 1	1888 888 88	18888888 11	88 188 11888	599999999984
11118 1118	8888 88888 8	88 18 111888	1888 888 11	18888888 88	88 188 118 1	88 111 888 8	188 11118 8	188 8 1188	88 11 8888 18	599999999984
1118 111188	88888 8888 1	88 118 1188	188 8888 18	188 1888 8	88 11111 8 8	88 188888 88	18888 118 1	1888 111888	88 18 888 11	599999999984
11111 8888 8	8888 111118	88 188888 11	18888 11888	1888 1118 1	88 18 888 88	88 118 118 8	188 8888 8	188 1888 18	88 11111 88	599999999984
888 8 88 18	1188 1188 8	18 1888 888	88 8 8 18 11	88 8 1118 88	18 1188 8 1	18 88 18 8	88 88888 8 8	88 88 88 88	18 8 118 18	599999999984
888 111188 1	118 188 188	18 88 18 18	88 88888 88	88 88 88 8 8	18 8 118 8	18 1888 8 1	88 8 8 18 88	88 8 1118 11	18 1188 888	599999999984
888 88 188 8	1188888 18	18 11118 11	88 8 188 888	88 8 888 8 1	18 18 18 88	18 8 88 8 8	88 88 118 8	88 888 18 18	18 8888 88	599999999984
888 1888 188	118 8 188 1	18 8 88 88	88 88 118 18	88 888 18 8	18 8888 8 8	18 11118 88	88 8 188 8 1	88 8 888 888	18 18 18 11	599999999984
1188888 111	888 88 18888	88 8 188 8 8	18 11118 8	18 18 18 18	88 8 888 88	88 88 118 11	18 8 88 888	18 8888 8 1	88 888 18 88	599999999984
118 8 1888	888 1888 118	88 88 118 88	18 8 88 8 1	18 8888 888	88 888 18 11	88 8 188 88	18 11118 18	18 18 18 8	88 8 888 8 8	599999999984
1188 118888	888 8 88 111	88 8 8 18 8	18 1888 8 8	18 1188 88	88 8 1118 18	88 88888 888	18 88 18 11	18 8 118 88	88 88 88 8 1	599999999984
118 188 118	888 1111888	88 88888 8 1	18 88 18 88	18 8 118 11	88 88 88 888	88 8 8 18 18	18 1888 88	18 1188 8 8	88 8 1118 8	599999999984
88 88 888 8	18 8 1118 8	118 8888888	888 18 11111	888 11111 88	118 18888 1	11888 1118	888 888888 8	888 8 88888	1188 111118	599999999984
88 8 11118 1	18 11888 88	11888 11118	888 8888888	888 8 8888 8	1188 11118	118 88888 1	888 18 11188	888 1111111	118 1888888	599999999984
88 888 118 8	18 88888 8	118 1111111	888 11888888	888 188888 1	118 8 11188	1188 8888 8	888 8 11118	888 88 11118	1188888888	599999999984
88 8 8888 88	18 18 118 1	1188 88888	888 8 111118	888 88 1118	118888888 8	118 1111188	888 118888 1	888 18888888	118 8 11111	599999999984
18 88888 11	88 888 11888	888 118888 8	118 111118	118 8 11118	888 1888888	888 8 111111	1188 888888	118888888 1	888 88 11188	599999999984
18 18 1188	88 8 8888 18	888 8 111188	1188 8888 1	11888888888	888 88 11111	888 1188888	118 1111118	118 8 1118	888 188888 8	599999999984
18 8 111888	88 88 888 11	888 18 1118	118 88888 8	118 188888	888 11111118	888 88888888	11888 11111	1188 111188	888 8 8888 1	599999999984
18 11888 18	88 8 111188	888 888888 1	11888 11188	1188 111111	888 8 888888	888 18 11118	118 888888	118 18888 8	888 1111118	599999999984
88 18 88 88	1888 118 18	1111 888 188	8888 8 188 1	8888 1118888	11111 88 111	11188 1888	88888888 118	88888 88 18	1118 1188 8	599999999984
88 111118 11	188 188 888	11188 188 8	88888888 18	88888 88 118	1118 11888	1111888 111	8888 8 18888	8888 11188 1	1111 88 188	599999999984
88 188 18 18	1888888 88	111111188 1	8888 188 188	8888 888 111	11118 18888	1118 88 118	88888 11888	888888 188 8	1118888 18	599999999984
88 11888 888	188 8 18 11	1118 88 18	88888 1188 8	888888 1888	1118888 118	11111118888	8888 188 111	8888 888 188	11118 188 1	599999999984
1888888 8 1	88 188 18 88	8888 188 118	1111111888	11118 188 8	8888 888 18	88888 1188 1	1118 88 188	1118888 111	888888 18888	599999999984
188 8 18 8	88 11888 8 8	88888 118888	1118 88 111	1118888 188	888888 188 1	8888 188 18	111111188 8	11118 1888	8888 888 118	599999999984
1888 118 88	88 18 88 8 1	8888 8 1888	1111888 118	11111 88 18	8888 11188 8	88888888 188	11188 188 11	1118 118888	88888 88 111	599999999984
188 188 8 8	88 111118 8	88888888 111	11188 18888	1118 1188 1	88888 88 188	8888 8 188 8	1111888 18	1111888 118	8888 111888	599999999984
599999999984	599999999984	599999999984	599999999984	599999999984	599999999984	599999999984	599999999984	599999999984	599999999984	599999999984
23	24	25	26	27	28	29	30	31	32	

It is based on 2 by 2 combinations of 32 numbers of 6-digits. These are divided in four groups of eight each with equal sums. See below:

$$\begin{aligned}
 A &:= 111111 + 111118 + 111181 + 111188 + 888811 + 888818 + 888881 + 888888 = 3999996 \\
 B &:= 111811 + 111818 + 111881 + 111888 + 888111 + 888118 + 888181 + 888188 = 3999996 \\
 C &:= 118111 + 118118 + 118181 + 118188 + 881811 + 881818 + 881881 + 881888 = 3999996 \\
 D &:= 118811 + 118818 + 118881 + 118888 + 881111 + 881118 + 881181 + 881188 = 3999996
 \end{aligned}$$

These 32 numbers are chosen in little different way. Symmetrizing compliments of 1 and 8 are chosen in pairs as:  $\{111118, 888881\}$ ,  $\{111181, 888818\}$ ,  $\{111188, 888811\}$ ,  $\{111111, 888888\}$ ,  $\{111811, 888188\}$ ,  $\{111818, 888181\}$ ,  $\{111881, 888118\}$ ,  $\{111888, 888111\}$ ,  $\{118111, 881888\}$ ,  $\{118118, 881881\}$ ,  $\{118181, 881818\}$ ,  $\{118188, 881811\}$ ,  $\{118811, 881188\}$ ,  $\{118818, 881181\}$ ,  $\{118881, 881118\}$  and  $\{118888, 881111\}$ .

Let's see the **upside down** and **mirror looking** versions of the numbers given in above four groups  $A$ ,  $B$ ,  $C$  and  $D$ :

$$\begin{aligned}
 A &:= 111111 + 811111 + 181111 + 881111 + 118888 + 818888 + 188888 + 888888 = 3999996 \\
 B &:= 118111 + 818111 + 188111 + 888111 + 111888 + 811888 + 181888 + 881888 = 3999996 \\
 C &:= 111811 + 811811 + 181811 + 881811 + 118188 + 818188 + 188188 + 888188 = 3999996 \\
 D &:= 118811 + 818811 + 188811 + 888811 + 111188 + 811188 + 181188 + 881188 = 3999996
 \end{aligned}$$

Even though the numbers are different but the sum is same. Thus it is a **universal bimagic square** with **magic** and **bimagic sums**:

$$S_{32 \times 32} := 15999999999984 = 1000000 \times 15999984 + 15999984,$$

and

$$Sb_{32 \times 32} := 12743595959580743595959600$$

where

$$\begin{aligned}
 11999988 &:= 111111 + 111118 + 111181 + 111188 + 888811 + 888818 + 888881 + 888888 + \\
 &\quad + 111811 + 111818 + 111881 + 111888 + 888111 + 888118 + 888181 + 888188 + \\
 &\quad + 118111 + 118118 + 118181 + 118188 + 881811 + 881818 + 881881 + 881888 + \\
 &\quad + 118811 + 818811 + 188811 + 888811 + 111188 + 811188 + 181188 + 881188
 \end{aligned}$$

Moreover the above 16 magic squares of order 8 are **pandiagonal** and **bimagic** with equal **magic sums**:

$$S_{8 \times 8} := 3999999999996 = 1000000 \times 3999996 + 3999996,$$



### 3 Final Comments

- (i) All the blocks of order 4 are of equal magic sums given in **magic squares** of orders 20, 24, 28 and 32.
- (ii) All the blocks of order 6 are of equal magic sums given in **magic squares** of orders 18, 24 and 30.
- (iii) All the blocks of order 8 are of equal magic sums given in **magic squares** of orders 24 and 32. In case of order 24 the magic square is **semi-bimagic**, and in case of order 32, the **magic square** is **bimagic**.
- (iv) All the blocks of order 5 are of equal magic sums given in **bimagic squares** of orders 25.
- (v) Whole the work is 10-digits distributions of two numbers 1 and 8. There are only three magic squares of orders 24, 25 and 32 are with 12-digits distributions of two numbers 1 and 8. In the first case, i.e., **magic square** of order 24 it is **semi-bimagic**, while other two cases of orders 25 and 32, the **magic squares** are **bimagic squares**. These three magic squares are given in Examples 2.14, 2.16 and 2.27.

### 4 Appendix

Let's consider two letters *a* and *b*. Below are the combinations of these two letters:

2 – *digits* :  $2^2 := 4$

*aa, ab, ba, bb*

3 – *digits* :  $2^3 := 8$

*aaa, aab, aba, abb, baa, bab, bba, bbb*

4 – *digits* :  $2^4 := 16$

*aaaa, aaab, aaba, aabb, abaa, abab, abba, abbb, baaa, baab, baba, babb, bbaa, bbab, bbba, bbbb*

5 – *digits* :  $2^5 := 32$

*aaaaa, aaaab, aaaba, aaabb, aabaa, aabab, aabba, aabbb, abaaa, abaab, ababa, ababb, abbaa, abbab, abbba, abbbb, baaaa, baaab, baaba, baabb, babaa, babab, babba, babbb, bbaaa, bbaab, bbaba, bbabb, bbbba, bbbab, bbbba, bbbbb*

aaaaaa,aaaaab,aaaaba,aaaabb,aaabaa,aaabab,aaabba,aaabbb,aabaaa,aababa,aababb,  
aabbaa,aabbab,aabbba,aabbbb,abaaaa,abaaab,abaaba,abaabb,ababaa,ababab,ababba,  
ababbb,abbaaa,abbaab,abbaba,abbabb,abbbaa,abbbab,abbbba,abbbbb,baaaaa,baaaab,  
baaaba,baaabb,baabaa,baabab,baabba,baabbb,babaaa,babaab,bababa,bababb,babbba,  
babbab,babbba,babbbb,bbaaaa,bbaaab,bbaaba,bbaabb,bbabaa,bbabab,bbabba,bbabbb,  
bbbbaa,bbbaab,bbbaba,bbbabb,bbbbaa,bbbbab,bbbbba,bbbbbb

aaaaaaaa,aaaaaab,aaaaaba,aaaaabb,aaaabaa,aaaabab,aaaabba,aaaabbb,aaabaaa,  
aaabaab,aaababa,aaababb,aaabbaa,aaabbab,aaabbba,aaabbbb,aabaaaa,aabaaab,  
aabaaba,aabaabb,aababaa,aababab,aababba,aababbb,aabbaaa,aabbaab,aabbaba,  
aabbabb,aabbbaa,aabbbab,aabbbbba,aabbbbb,abaaaa,abaaaab,abaaaba,abaaabb,  
abaabaa,abaabab,abaabba,abaabbb,ababaaa,ababaab,abababa,abababb,ababbaa,  
ababbab,ababbba,ababbbb,abbaaaa,abbaaab,abbaba,abbabb,abbabaa,abbabab,  
abbabba,abbabbb,abbbaaa,abbbaab,abbbaba,abbbabb,abbbbba,abbbbab,abbbbba,  
abbbbbbb,baaaaa,baaaaab,baaaaba,baaaabb,baaabaa,baaabab,baaabba,baaabbb,  
baabaaa,baabaab,baababa,baababb,baabbaa,baabbab,baabbba,baabbbb,babaaa,  
babaaab,babaaba,babaabb,bababaa,bababab,bababba,bababbb,babbaaa,babbaab,  
babbaba,babbabb,babbbba,babbbab,babbbbba,babbbbb,bbaaaa,bbaaaab,bbaaaba,  
bbaaabb,bbabaaa,bbaabab,bbaabba,bbaabbb,bbabaaa,bbabaab,bbababa,bbababb,  
bbabbba,bbabbab,bbabbba,bbabbbb,bbbbaaa,bbbbaab,bbbaba,bbbabb,bbbabaa,  
bbbabab,bbbabba,bbbabbb,bbbbaaa,bbbbaab,bbbbaba,bbbbabb,bbbbbaa,bbbbbab,  
bbbbbbba,bbbbbbb

aaaaaaaa,aaaaaaab,aaaaaaba,aaaaaabb,aaaaabaa,aaaaabab,aaaaabba,aaaaabbb,  
aaaabaaa,aaaabaab,aaaababa,aaaababb,aaaabbaa,aaaabbab,aaaabbba,aaaabbbb,



*aaabaaaa, aaabaaab, aaabaaba, aaabaabb, aaababaa, aaababab, aaababba, aaababbb,  
aaabbaaa, aaabbaab, aaabbaba, aaabbabb, aaabbbaa, aaabbbab, aaabbbba, aaabbbbb,  
aabaaaaa, aabaaaab, aabaaaba, aabaaabb, aabaabaa, aabaabab, aabaabba, aabaabbb,  
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bbabbaaa, bbabbaab, bbabbaba, bbabbabb, bbabbbba, bbabbbbaab, bbabbbba, bbabbbbb,  
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bbbabaaa, bbbabaab, bbbababa, bbbababb, bbbabbaa, bbbabbab, bbbabbba, bbbabbbb,  
bbbbaaaa, bbbbbaab, bbbbbaaba, bbbbbaabb, bbbbabaa, bbbbabab, bbbbabba, bbbbabbb,*

*bbbbbaaa, bbbbaaab, bbbbabab, bbbbabbb, bbbbbbba, bbbbbbba, bbbbbbba, bbbbbbba*

$$8 - \textit{digits} : 2^9 := 512$$

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*ababbabba, ababbabbb, ababbbaaa, ababbbaab, ababbbaba, ababbbabb, ababbbbbaa,  
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bbaaaaaab, bbaaaaaba, bbaaaaabb, bbaaaaabaa, bbaaaaabab, bbaaaaabba, bbaaaaabbb,  
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 bbaabbbab, bbaabbbbba, bbaabbbbb, bbabaaaaa, bbabaaaab, bbabaaaba, bbabaaabb,  
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 bbabbbaab, bbabbbaba, bbabbbab, bbabbbaa, bbabbbbab, bbabbbbba, bbabbbbbb,  
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 bbbaaabbb, bbbaabaaa, bbbaabaab, bbbaababa, bbbaababb, bbbaabbaa, bbbaabbab,  
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 bbbabbbaa, bbbabbbab, bbbabbba, bbbabbbb, bbbbaaaaa, bbbbaaaaab, bbbbaaaba,  
 bbbbaaabb, bbbbaabaa, bbbbaabab, bbbbaabba, bbbbaabbb, bbbbabaaa, bbbbabaaab,  
 bbbbababa, bbbbababb, bbbbabbaa, bbbbabbab, bbbbabbbba, bbbbabbbb, bbbbbaaaa,  
 bbbbbaaab, bbbbbaaba, bbbbbaabb, bbbbbabaa, bbbbbabab, bbbbbabba, bbbbbabbb,  
 bbbbbbbaa, bbbbbbbaab, bbbbbbba, bbbbbbabb, bbbbbbbaa, bbbbbbbaab, bbbbbbba,  
 bbbbbbba

8 – digits :  $2^{10} := 1024$

aaaaaaaaa, aaaaaaaaab, aaaaaaaaaba, aaaaaaaaabb, aaaaaaabaa, aaaaaaabab, aaaaaaabba,  
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 aaaaaabbba, aaaaaabbbb, aaaaabaaaa, aaaaabaaab, aaaaabaaba, aaaaabaabb, aaaaababaa,  
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[illegible]

[illegible]



[illegible]

## 5 Author's Contributions to Magic Squares

The **item-wise** author's work on magic squares is as follows:

*i. Digital Numbers Magic Squares - [5, 6, 7, 8, 9, 10, 37];*

- ii. Block-Wise Construction of Bimagic Squares - [11];*
- iii. Connections with Genetic Tables and Shannon's entropy - [12];*
- iv. Selfie and Palindromic-type Magic Squares - [13, 28, 37];*
- v. Intervally Distributed and Block-Wise Magic Squares - [14, 15, 16, 28];*
- vi. Multi-digits and Number Patterns Magic Squares - [17, 27];*
- vii. Perfect Square Sum Magic Squares with Uniformity, Minimum Sum and Pythagorean Triples - [18, 19];*
- viii. Block-Wise Constructions of Magic and Bimagic Squares - [20, 21, 22, 23, 26, 29, 31];*
- ix. Magic Crosses: Repeated and Non Repeated Entries - [24];*
- x. Representations of Letters and Numbers With Equal Sums Magic Squares of Orders 4 and 6 - [25].*
- xi. Bordered Magic Squares - [32, 33, 34, 35, 36].*
- xii. 2-Digits Magic and Bimagic Squares - [38].*

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